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Abstract

APPLICABILITY OF THRUST AUGMENTING EJECTORS IN A SUPERSONIC CRUISE CONFIGURATION

by Wendell Scott Hertzelle

Chairperson of the Supervisory Committee: Professor Abraham Hertzberg
Department of Aeronautics and Astronautics

Thrust augmenting ejectors were analyzed by the author in an attempt to see if they could be used to provide a significant thrust increase over the baseline thrust of a primary core. This was done for the purpose of determining if leaving the ejectors open in the cruise configuration of the High Speed Civil Transport would lead to a thrust increase that would at least offset the weight of the ejectors themselves. If this was found to be true, then the fuel economy of the HSCT might be improved by leaving them open and not closing them during cruise.

In analyzing the ejectors, no assumptions were made regarding inlet nor outlet configurations, so an attempt was made to find the point of optimal thrust augmentation by varying secondary stream bypass Mach number and the amount of flow entrainment. Two solutions were found to each mixing scenario, one subsonic and the other supersonic. These two solutions were each analyzed and ones not satisfying the Second Law of Thermodynamics were eliminated. Analytic diffuser and bleed losses were also explored in the analysis of the ejector flow. Within the limitations of the assumptions discussed in this paper, appreciable thrust augmentations have been discovered over a large range of bypass Mach numbers and entrained mass flows. This lead the author to

the conclusion that ejectors warrant further research beyond a first order analysis, and serious thought should be put into leaving them open in cruise.

APPLICABILITY OF THRUST AUGMENTING
EJECTORS IN A SUPERSONIC CRUISE
CONFIGURATION

by

Wendell Scott Hertzelle

A thesis submitted in partial fulfillment of the
requirements for the degree of

Master of Science in Aeronautics and Astronautics

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the conclusion that ejectors warrant further research beyond a first order analysis, and serious thought should be put into leaving them open in cruise.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Number *Page*

GLOSSARY

Word. Bypass Mach Number - This is the Mach number of the secondary or ejector flow. It is a function of Pressure Recovery, P_r.

Word. Ejector - An ejector is a type of device that entrains or induces a slower mass flow around a high velocity primary flow.

Word. Entrain - This is the process of capturing a flow by translation through some medium or inducing secondary flow with a primary flow.

Word. High Speed Civil Transport (HSCT) - This is the proposed name for the supersonic passenger aircraft of the future.

Word. Mass Flow Ratio (μ = r) - This is the ratio of the secondary mass flow and the primary mass flow. This dimensionless number is the ratio of the individual flows given in kg/s.

Word. Pressure Recovery (P_r) - This is a parameter that is chosen and varied for purposes of solving the problem. It is the ratio of the static pressure over the stagnation pressure of the secondary flow. Its selection fixes bypass Mach number.

Word. Thrust Augmentation - This is the percent increase, or decrease as the case could be, in thrust of the ejector/primary combination over the thrust of the primary flow alone.

LIST OF ABBREVIATIONS

Abbreviation. α - area ratio

Abbreviation. γ - specific heat ratio (C_p/C_v)

Abbreviation. $\mu = r$ - mass flow ratio

Abbreviation. ρ - flow density

Abbreviation. A - area

Abbreviation. C_p - specific heat at constant pressure

Abbreviation. C_v - specific heat at constant volume

Abbreviation. $\dot{E}$ - energy flow

Abbreviation. F- thrust

Abbreviation. h_t - total enthalpy

Abbreviation. h- enthalpy

Abbreviation. K - dummy variable substituted for right hand side of equation (30) equivalent to momentum in.

Abbreviation. HSCT - High Speed Civil Transport

Abbreviation. J - dummy variable in Appendix B used in derivation of Alperin/Wu solution to the momentum equation

Abbreviation. lbm - pounds mass

Abbreviation. $\dot{m}$ - mass flow

Abbreviation. M - Mach number (velocity/local speed of sound)

Abbreviation. P - static pressure

Abbreviation. P_0 - total or stagnation pressure

Abbreviation. P_r - pressure recovery

Abbreviation. psia - pounds per square inch actual

Abbreviation. R - ideal gas constant [287 J/(kg·K)]

Abbreviation. s - specific entropy (entropy/unit mass)

Abbreviation. S - entropy

Abbreviation. ΔS - change in entropy

Abbreviation. T - static temperature

Abbreviation. T_0 - total or stagnation temperature

Abbreviation. v - velocity

Abbreviation. X - dummy variable equal to γM_6^2; used for simplification of mixing solution in Appendix B

PREFACE

During the fall of 1995, while taking an "self-structured" course in fluid mechanics at the University of Washington, the author and his class came upon the topic of ejectors and how they could be used for thrust augmentation. This topic was discussed in class as a comparison to ducted rockets, ramjets and other propulsion topics, but it was never applied directly to the topic of use on the High Speed Civil Transport.

The course advisor, Professor Abraham Hertzberg mentioned to the author that he had been working with another graduate student, Mr. Greg Williams, on the use of ejectors to increase thrust on the HSCT. According to Boeing, ejectors were being looked at for use in noise abatement during takeoff. By mixing the hot primary flow with a cooler secondary flow inside the ejector, the noise that resulted from that mixing could then be absorbed by noise abating material contained in the ejector itself, thus reducing takeoff noise.

Professor Hertzberg indicated that Boeing planned to leave the ejectors open during takeoff and the subsonic portion of the takeoff but then close the ejectors as cruise was reached. However, he felt that it would be worthwhile to consider leaving them open during cruise and see if some thrust augmentation could be realized. By doing so, some if not all of the weight of the ejectors themselves would be offset and therefore improve the fuel economy and the range of the aircraft itself.

Boeing was asked for data regarding the cruise configuration, the primary core engine data, and the ejector itself. Taking this data, Greg Williams compiled an initial spreadsheet that modeled the use of the ejectors in the supersonic cruise configuration at

Mach 2.4, and from this point, the author of this paper took over and expanded upon his research.

Modifications were made to the baseline spreadsheets to include diffuser losses, bleed losses and to calculate entropy changes. Also, further thought was given to the interpretation of the mixing results. All of these additions culminated in the creation of this paper and the results presented within.

ACKNOWLEDGMENTS

First of all, the author would like to acknowledge his advisor Professor Abraham Hertzberg for introducing him to this problem. By sharing this interesting problem with him, the author has been able to expand his understanding of fluid mechanics and open cycle propulsion, which will prove useful in his future Air Force career as a pilot.

Additionally, the author also wishes to acknowledge the help of Mr. Greg Williams, who initially began the research for this project. Without his initial work on the development of the spreadsheet used, the time necessary to complete this project would have been much longer. The work he completed was well done and served as a strong foundation upon which to expand.

Also, the assistance Boeing Corporation, who provided the primary core data, was greatly appreciated. Dr. Edwin Stear was the primary contact with Boeing, and special thanks is given to him. Other engineers from Boeing, whom the author would also like to thank, are Jan Syberg and Larry Clark. While they did not contribute to the project directly, they made the author recheck some of the work he had already done to ensure its viability.

DEDICATION

I wish to dedicate this thesis to my family, for all the support and love they have shown in everything I have done. I would like to especially dedicate this thesis to my grandfather, Aubray Hertzelle, and my mother, Beverly, without whom, I would never have made it to where I am today. Also, thank you Grandma for always being there to listen when I needed an ear.

INTRODUCTION

In order to provide an improvement in the design of the High Speed Civil Transport (HSCT), ejectors were analyzed to determine if they could be used for thrust augmentation. During takeoff, ejectors were planned to be used for purposes of noise abatement, and then closed during cruise. However, the author and his advisor wanted to look at what would occur if they were left open during the duration of the flight.

The choice of cruise location and speed were set by the design specifications provided by the Boeing Corporation. Their design, which was already in progress, provided the starting information needed to begin the problem analysis. After being provided data by Boeing, the author was able to begin his analysis of the applicability of ejectors for thrust augmentation.

CHAPTER 1: BACKGROUND AND PROBLEM SETUP

1.1: PROBLEM ORIGIN

In order to design a supersonic commercial transport, the aircraft must conform to a rigid noise regulation standard, FAR Part 36, Stage III. The Boeing Company has proposed to use ejectors on their High Speed Civil Transport (HSCT) for purposes of noise suppression. The phenomena of noise suppression will not be discussed in this thesis; however, the thrust augmentation properties of ejectors will be covered.

Boeing plans to stow the ejectors in cruise configuration, but through analysis it can be shown that a significant increase can be achieved over the baseline thrust if the ejectors are not stowed. The thrust increase could be used to offset the weight of the ejector itself (which Boeing stated is on the same order as the engine's weight).

1.2: SOLVING THE PROBLEM

1.2.1: BOEING DATA

In order to analyze the thrust augmentation properties of ejectors, the engine that Boeing proposes to use for the HSCT was used.[1] According to the data provided about the engine, it would have the following fixed characteristics:

Table 1: Engine Characteristics as Supplied by Boeing

Mass Flow; $\dot{m} = 198.6$ kg/s $\equiv \dot{m}_4$
Stagnation Temperature, $T_0 = 1265$ K $\equiv T_{04}$
Stagnation Pressure, $P_0 = 323{,}780$ Pa $\equiv P_{04}$

Also, the analysis was conducted at 50,000 feet (15,240 m) in standard atmosphere, with a flight Mach number of 2.4. Values for ambient conditions were given in the Boeing data, but Introduction to Flight by Anderson was referenced to check the accuracy of their values.[2] So, in summary, the free stream conditions are:

Table 2: Free Stream Conditions as Supplied by Boeing

Altitude = 50,000 ft = 15240 m
Density, $\rho_\infty = 0.1865$ kg/m$^3 \equiv \rho_1$
Ambient Temperature, $T_\infty = 216.7$ K $\equiv T_1$
Ambient Pressure, $P_\infty = 11{,}597$ Pa $\equiv P_1$
Flight Mach Number, $M_\infty = 2.4 \equiv M_1$

1.2.2: Problem Setup

In order to analyze the primary flow and the entrained secondary mass flow, the following stations were defined:

1. Ambient Conditions

2. Just prior to Diffuser Throat

3. Just after the Diffuser Throat

4. Primary Stream Flow

5. Ejector (Secondary Stream) Flow

6. Mixing Plane

7. Mixed Flow expanded to Ambient Conditions

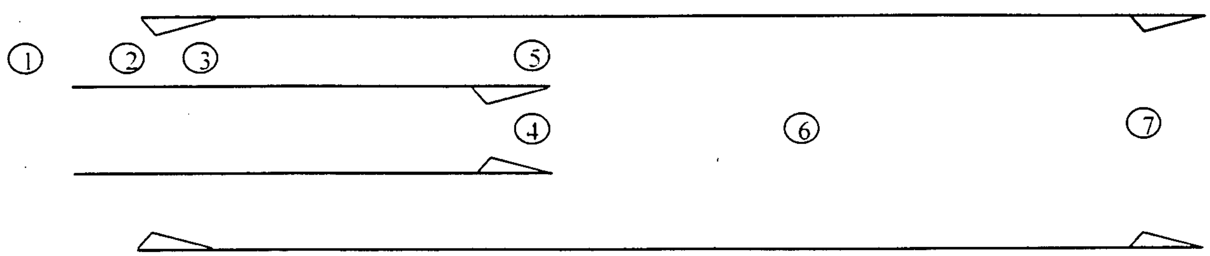

Figure 1: Diagram of Station Numbers

These stations were initially designated by Mr. Greg Williams, a former graduate student at the University of Washington, when he created the initial spreadsheet that was used to analyze this problem. So, to maintain consistency with previous work, the same convention has been adopted.

To simplify the analysis, the following assumptions were made:

1. There is no shock in the secondary stream diffuser.

2. The primary and secondary flows are perfectly mixed at station 6.

3. The static pressures at stations 4 and 5 are equal (i.e. the primary flow, station 4, completely expands).

4. Perfect gas relations hold, and all fluids are compressible.

5. The specific heat and specific heat ratios for the combustion products in stream 4 are identical to air, so $\gamma = 1.4$ and $R = 287.0$ J/(kg $\cdot$ K).

6. Air is inviscid.

7. Flow is steady.

8. Mixing is completed (the flow is solved) in a constant area (straight tube) duct.

9. The surfaces of the system are adiabatic.

Two parameters that will be varied to analyze the ejector phenomena are entrained mass flow ratio and bypass Mach number. Mass flow ratio μ is defined as

$$\mu = \frac{\dot{m}_5}{\dot{m}_4} \tag{1}$$

and bypass Mach number is determined from the pressure ratio

$$P_r = \frac{P_5}{P_{05}} \tag{2}$$

Due to the isentropic relations for a perfect gas, P_{05} is simply related to P_5 by specific heat ratio (constant) and the Mach number of the secondary flow. Thus, secondary flow Mach number (bypass Mach number), is the only variable relating P_5 to P_{05}.

1.2.3: Free Stream Equations

First of all, one must know the stagnation conditions for the free stream. From the definition of Mach number we have

$$v_1 = M_1\sqrt{\gamma R T_1} \tag{3}$$

From Table 2 we know M_1 = 2.4 and T_1 = 216.7 K; since the fluid is air, γ = 1.4, and R = 287 J/(kg · K), so this gives

v_1 = 706.6 m/s

According to the stagnation relations for temperature

$$T_{01} = T_1 \cdot \left(1 + \frac{\gamma - 1}{2} M_1^2\right) \tag{4}$$

Substituting in the above value of T_1 from Table 2 one gets

T_{01} = 466.3 K

The isentropic relations also give

$$\frac{T_1}{T_{01}} = \left(\frac{P_1}{P_{01}}\right)^{\frac{\gamma - 1}{\gamma}} \tag{5}$$

solving for P_{01} one gets

$$P_{01} = P_1 \left(\frac{T_{01}}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} \tag{6}$$

substituting in the previously calculated values gives

P_{01} = 169,500 Pa

1.2.4: DIFFUSER CONDITIONS

For the preliminary setup of the spreadsheet, the conditions of station 2 and 3 are assumed to be the same. Later, a straight percentage loss in total pressure will be assumed to model diffuser losses. But for the initial setup, there is no loss in total pressure P_0 between station 1 and 5.

1.2.5: EJECTOR (SECONDARY FLOW) CONDITIONS

For the ejector (secondary flow) it was just stated that its stagnation conditions would be assumed to be the same as the freestream. Therefore,

$$P_{05} = P_{01} \tag{7}$$

then rewriting (2)

$$P_5 = P_r\, P_{05} \tag{8}$$

solving for Mach number from (4) and (5), and using (2), but with subscript 5 instead of 1, bypass Mach number becomes

$$M_5 = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{1}{P_r}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right]} \tag{9}$$

again the stagnation relations for the ejector gives {similar to (5), except with subscript 5 not 1}

$$T_5 = T_{05}\cdot\left(P_r\right)^{\frac{\gamma - 1}{\gamma}} \tag{10}$$

by definition of Mach number

$$v_5 = M_5 \sqrt{\gamma\, R\, T_5} \tag{11}$$

from the perfect gas relation

$$\rho_5 = \frac{P_5}{R\, T_5} \tag{12}$$

by the previously stated definition of μ, and given $\dot{m}_4$ is already fixed

$$\dot{m}_5 = \mu\ \dot{m}_4 \tag{13}$$

finally from the definition of mass flow, and the previous equation

$$A_5 = \mu\ \frac{\dot{m}_4}{\rho_5\ v_5} \tag{14}$$

1.2.6: PRIMARY FLOW CONDITIONS

For the primary flow, it was previously stated that the flow would be completely expanded; therefore, it was expanded to match its static pressure with that of the secondary flow; so

$$P_4 = P_r\, P_{05} = P_5 \tag{15}$$

Given that P_{04} is fixed from the Boeing data, and using the just calculated P_4, M_4 can be calculated from

$$M_4 = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_{04}}{P_4}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right]} \tag{16}$$

given that T_{04} is fixed by the engine data. T_4 can be calculated from the isentropic relation to pressure

$$T_4 = T_{04}\left(\frac{P_4}{P_{04}}\right)^{\frac{\gamma - 1}{\gamma}} \tag{17}$$

primary flow velocity is easily determined from the definition of Mach number

$$v_4 = M_4\sqrt{\gamma\, R\, T_4} \tag{18}$$

density comes from the perfect gas relation

$$\rho_4 = \frac{P_4}{R\, T_4} \tag{19}$$

finally, A_4 is also easily determined from the definition of mass flow

$$A_4 = \frac{\dot{m}_4}{\rho_4\, v_4} \tag{20}$$

1.2.7: Flow Mixing

In order to complete the analysis, the two flows must mix. It was assume that the two flows have mixed completely and perfectly by the time they reach station six. Beyond the losses incurred for mixing itself, no additional frictional losses were accounted for in the process. Also, the distance needed to complete mixing is not determined. To reiterate assumption 8, one item to note is that the mixing is accomplished in a constant area duct,

which has an area given as the sum of the primary and secondary flow areas, as previously calculated, so

$$A_6 = A_4 + A_5 \tag{21}$$

1.2.7.1: Continuity

To solve the mixing problem, the equations of continuity, momentum and energy are needed.[3] Continuity for a perfect gas states that

$$\dot{m}_6 = \dot{m}_4 + \dot{m}_5 = \dot{m}_4\,(1 + \mu) \tag{22}$$

1.2.7.2: Energy

Next, conservation of energy states that the energy flow into the mixing chamber equals the energy flow out, so

$$\dot{E}_6 = \dot{E}_4 + \dot{E}_5 \tag{23}$$

In general, energy flow is equal to the following according to the definition of total enthalpy

$$\dot{E} = \dot{m}\,h_t \tag{24}$$

total enthalpy is defined as

$$h_t = h + {}^1/_2\,v^2 = C_pT + {}^1/_2\,v^2 \tag{25}$$

substituting into (24) and utilizing the definition of velocity as defined by Mach number one gets

$$\dot{E} = \dot{m}\left[C_p T + \frac{1}{2} M^2 \gamma R T\right] \tag{26}$$

since $\gamma = C_p/C_v$

$$\dot{E} = \dot{m}\, C_p T\left[1 + \frac{M^2 R}{2\, C_v}\right] \tag{27}$$

plus since $R = C_p - C_v$, and remembering the definition of stagnation temperature similar to equation (4) gives

$$\dot{E} = \dot{m}\, C_p T\left[1 + \frac{\gamma - 1}{2} M^2\right] = \dot{m}\, C_p T_0 \tag{28}$$

substituting this definition of energy flow (28) into equation (23) and canceling C_p since it is assumed to be constant gives

$$\dot{m}_6 T_{06} = \dot{m}_4 T_{04} + \dot{m}_5 T_{05} = \dot{m}_4 (T_{04} + \mu T_{05}) \tag{29}$$

this can easily be solved for T_{06}.

1.2.7.3: Momentum

This leaves the momentum equation to be solved, which states that for a straight tube solution

$$\dot{m}_6 v_6 + P_6 A_6 = \{\dot{m}_4 v_4 + P_4 A_4 + \dot{m}_5 v_5 + P_5 A_5\} \tag{30}$$

Since all the values on the right hand side of the equation are known, for simplicity call that value K. (i.e. $K = \dot{m}_4 v_4 + P_4 A_4 + \dot{m}_5 v_5 + P_5 A_5$) Mixed mass flow $\dot{m}_6$ is given by equation (22), and A_6 is given by (21), but P_6 and v_6 are still variables in question. To

solve this dilemma, first rewrite velocity in terms of Mach number and using the perfect gas law along with the definition of mass flow, P_6 may be conveniently eliminated.

First of all, since mass flow is $\rho A v$ and $\rho = P/(R\,T)$, A may be solved for and is equal to

$$A = \frac{\dot{m}\, R\, T}{P\, v} \tag{31}$$

substituting in the definition for velocity as defined by Mach number (Equation (3) without subscript 1), one gets

$$A = \frac{\dot{m}\, R\, T}{P\, M \sqrt{\gamma\, R\, T}} = \frac{\dot{m}}{P\, M}\sqrt{\frac{R\, T}{\gamma}} \tag{32}$$

Thus, putting (32) back into (30), but with subscript 6, and remembering that the right hand side is now K, one gets

$$\dot{m}_6\, v_6 + \frac{P_6\, \dot{m}_6}{P_6\, M_6}\sqrt{\frac{R\, T_6}{\gamma}} = K \tag{33}$$

again utilizing the definition of Mach number and also using the stagnation temperature relation for T_6 (similar to Equation (4), but with subscript 6)

$$\dot{m}_6 \sqrt{\frac{T_{06}}{1 + \frac{\gamma - 1}{2} M_6^2}} \left[M_6 \sqrt{\gamma\, R} + \frac{1}{M_6}\sqrt{\frac{R}{\gamma}} \right] = K \tag{34}$$

Equation (34) is then simply a function of the mixing plane Mach number M_6, so utilizing the Newton solver, which comes with the EXCEL spreadsheet, a solution was found. There are two possible solutions to the mixing plane Mach number. These are the so called first and second solutions. The first solution is the subsonic solution, and the

second solution is the supersonic one. It is possible to get only one solution (i.e., it mechanically chokes, but this will be discussed further in the analysis of the results). For an alternate solution to this problem, where M_6 is solved for explicitly, see Appendix B.[4] In this appendix, an analytic solution for M_6 is shown and the conditions under which mechanical choking can occur are summarized.

1.2.8: THRUST GENERATION AND FLOW EXPANSION

Now that the flow has mixed, it must be "expanded" back to ambient pressure. The exact process that the expansion/compression cycle would take is dictated by the end of mixing Mach number. There are three possible ways to bring it back to local ambient pressure. The three possibilities are, a converging duct, a diverging duct, and finally, a converging then diverging duct. One of these three will allow the mixed flow to "expand" back to the current ambient pressure.

Normally, the thrust equation for an open cycle engine takes the following form [5] (Which would be the case if the flow were not "expanded" through a system of ducts and was allowed to exit at the mixing cross sectional area).

$$F = \dot{m}_6 (v_6 - v_\infty) + A_6 (P_6 - P_\infty) \tag{35}$$

However, since the mixed flow will be expanded back to ambient pressure, then the pressure at the "nozzle" exit will be equal to P_∞, so the second term is zero. Recalling from Figure 1, that the nozzle exit plane is 7, thrust then becomes

$$F = \dot{m}_7 (v_7 - v_\infty) \tag{36}$$

Now it is just a matter of determining v_7, since by continuity, $\dot{m}_6 = \dot{m}_7$, and $v_\infty = v_1$ is already known. To determine v_7, we need to know P_{07}, so we can calculate M_7 and then calculate v_7 from there. The formulas needed to do this are

$$P_{07} = P_{06} = P_6 \left(1 + \frac{\gamma - 1}{2} M_6^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{37}$$

$$P_7 = P_\infty \tag{38}$$

$$M_7 = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_{07}}{P_7}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right]} \tag{39}$$

and since the process is an isentropic expansion, stagnation temperature is conserved, so

$$T_{07} = T_{06} \tag{40}$$

$$v_7 = \sqrt{\frac{\gamma\, R\, T_{07}\, M_7^2}{1 + \frac{\gamma - 1}{2} M_7^2}} \tag{41}$$

Now, having calculated v_7, the thrust produced from the primary flow/ejector combination can be calculated using (36). The form of v_7 in Equation (41) is cumbersome to look at, but it is implemented that way in the spreadsheet to avoid the calculation of T_7.

1.2.9: Entropy Production

Knowing there are two possible Mach numbers for each solution, the natural question is, "Which one will be the one that will naturally occur?". This question may be answered

by looking to the Second Law of Thermodynamics.

In any thermodynamic process, there is an associated entropy change. In all natural processes, this change occurs as an entropy increase. Thus, if the process would have a "negative" change in entropy, it would not be naturally realizable. So, in looking at the ejector problem, the entropy change for each given mixing scenario must be considered.

From the well known thermodynamic result, for an adiabatic system[6]

$$s = C_p \ln T - R \ln P + \text{const} \tag{42}$$

one can conclude that the total entropy for an open system is

$$S = \dot{m}\left(C_p \ln T - R \ln P + \text{const}\right) \tag{43}$$

However, since we are assuming a thermally and calorically perfect gas, the constant will be dropped assuming the same thermodynamic reference for all gases.

The entropy of the ejector system is comprised of the entropy of the primary flow and the secondary flow (stations 4 and 5 respectively). Then entropy change for each is then

Secondary Flow

$$\Delta S_2 = S_6 - S_5 \approx \dot{m}_5(C_p \ln T_6 - R \ln P_6) - \dot{m}_5(C_p \ln T_5 - R \ln P_5) = \dot{m}_5(C_p \ln\frac{T_6}{T_5} - R \ln\frac{P_6}{P_5})$$

Primary Flow

$$\Delta S_1 = S_6 - S_4 \approx \dot{m}_4(C_p \ln T_6 - R \ln P_6) - \dot{m}_4(C_p \ln T_4 - R \ln P_4) = \dot{m}_4(C_p \ln\frac{T_6}{T_4} - R \ln\frac{P_6}{P_4})$$

So the total entropy change is

$$\Delta S_{tot} = \Delta S_1 + \Delta S_2 = \dot{m}_4(C_p \ln\frac{T_6}{T_4} - R \ln\frac{P_6}{P_4}) + \dot{m}_5(C_p \ln\frac{T_6}{T_5} - R \ln\frac{P_6}{P_5}) \quad (44)$$

Referencing it to the primary flow by dividing through by $\dot{m}_4 \cdot R$, one gets

$$\frac{\Delta S_{tot}}{\dot{m}_4 R} = \frac{\Delta S_1 + \Delta S_2}{\dot{m}_4 R} = \frac{C_p}{R}\ln\frac{T_6}{T_4} + \mu\frac{C_p}{R}\ln\frac{T_6}{T_5} - \ln\frac{P_6}{P_4} - \mu\ln\frac{P_6}{P_5} \quad (45)$$

Remembering $\gamma = C_p/C_v$ and $R = C_p - C_v$ ones gets

$$\frac{\Delta S_{tot}}{\dot{m}_4 R} = \frac{C_p}{C_p - C_v}\ln\frac{T_6}{T_4} + \mu\frac{C_p / C_v}{R / C_v}\ln\frac{T_6}{T_5} - \ln\frac{P_6}{P_4} - \mu\ln\frac{P_6}{P_5}$$

$$\frac{\Delta S_{tot}}{\dot{m}_4 R} = \frac{\gamma}{\gamma - 1}\ln\frac{T_6}{T_4} + \mu\frac{\gamma}{\gamma - 1}\ln\frac{T_6}{T_5} - \ln\frac{P_6}{P_4} - \mu\ln\frac{P_6}{P_5} \quad (46)$$

But since the primary flow is fully expanded, and $P_4 = P_5$

$$\frac{\Delta S_{tot}}{\dot{m}_4 R} = \frac{\gamma}{\gamma - 1}\ln\frac{T_6}{T_4} + \mu\frac{\gamma}{\gamma - 1}\ln\frac{T_6}{T_5} - (1 + \mu)\ln\frac{P_6}{P_4} \quad (47)$$

Therefore, by implementing this non-dimensionalized total entropy check into the spreadsheet analysis, it can be determined what solutions will be physically and thermodynamically consistent (i.e., $\Delta S_{tot} \geq 0$)

CHAPTER 2: INITIAL ANALYSIS

2.1: SPREADSHEET IMPLEMENTATION

By using the equations from the previous chapter, a spreadsheet was created using Microsoft EXCEL 5.0. One sheet was dedicated to each value of P_r, and is labeled with the appropriate value of bypass Mach number. On each sheet, mass flow ratio was varied from 0.0 to 2.0 by steps of 0.1. Then, after solving the momentum equation, and solving the expansion, thrust for each mass flow ratio was compared to the baseline ($\mu = 0.0$) on each sheet and was expressed as a percent increase or decrease.

Finally, all the data on thrust from each sheet ($P_r = 0.9 - 0.4$, or $M_5 = 0.39 - 1.22$), was combined on one sheet plotting bypass Mach number versus mass flow ratio versus percent thrust increase. To determine overall thrust augmentation, all values were compared to value of thrust calculated for mass flow ratio equal to 0.0. This is simply the thrust of the primary core without any additional ejector flow.

An example of one of the sheets produced by EXCEL is

Table 3: Worksheet for $P_r = 0.9003$

Ejector Calculations **Mach = 2.4** **Altitude = 50,000 ft = 15240 meters**

P_r=	0.9003								
Gas Properties		**Station 4 Calc's**		**Station 5 Calc's**		**Station4**		**Station5**	
R	287	P_{04}	323800	P_{05}	168327.	massflow	198.6	density	1.16678
γ	1.4	P_4	151545.	P_5	151545.	pressure	151545.3	pressure	151545.
U_1	708.183	M_4	1.10057	M_5	0.39027	velocity	703.9863	velocity	166.420
T_1	216.7	T_4	1018.31	T_{05}	466.338	T_{04}	1265	T_{05}	466.338
P_1	11597	V_4	703.986	T_5	452.552	Area	0.544045		
P_{01}	169548.	ρ_4	0.51853	V_5	166.420				
T_{01}	466.338	A_4	0.54404	ρ_5	1.16678				

μ	m_6	T_{06}	T_6	A_5	K	M_6	Solver	u_6	A_6	P_6	P_{06}	M_7	u_7	$\Delta s/m_4 R$	Thrust	%Incre
															Expand to P_1	Expand to P_1
0.0	198.6	1265	1018.31	0	1119.13	1.100573	-1.4E-08	703.986	0.54404	151545.	323800	2.81864	1248.90	3.28E-11	107386.7	0.00
0.1	218.46	1192.39	1057.34	0.10227	1103.47	0.79913	2.27E-13	520.872	0.64632	196920.	299914.	2.76846	1204.05	0.14057	108328.4	0.88
0.2	238.32	1131.89	1019.07	0.20455	1090.42	0.743991	4.55E-13	476.075	0.7486	195579.	282434.	2.72928	1166.47	0.26474	109218.8	1.71
0.3	258.18	1080.69	982.720	0.30683	1079.37	0.70603	0	443.652	0.85087	192896.	269016	2.69759	1134.41	0.37602	110044.4	2.47
0.4	278.04	1036.81	949.716	0.40911	1069.91	0.677147	2.27E-13	418.297	0.95315	190079.	258406.	2.67146	1106.72	0.47650	110809.5	3.19
0.5	297.9	998.779	920.073	0.51138	1061.70	0.653998	-4.5E-13	397.642	1.05543	187435.	249813.	2.64952	1082.53	0.56782	111518.7	3.85
0.6	317.76	965.501	893.485	0.61366	1054.53	0.634828	0	380.368	1.15771	185039.	242713.	2.63085	1061.20	0.65128	112176.4	4.46
0.7	337.62	936.139	869.590	0.71594	1048.19	0.618583	0	365.645	1.25998	182893.	236749.	2.61475	1042.24	0.72793	112787.1	5.03
0.8	357.48	910.039	848.044	0.81822	1042.56	0.60458	0	352.913	1.36226	180976.	231669	2.60073	1025.27	0.79862	113355	5.56
0.9	377.34	886.686	828.543	0.92049	1037.53	0.592347	0	341.774	1.46454	179262.	227290.	2.58840	1009.99	0.86406	113884	6.05
1.0	397.2	865.669	810.826	1.02277	1032.99	0.581544	0	331.933	1.56682	177725	223477.	2.57748	996.142	0.92485	114377.5	6.51
1.1	417.06	846.653	794.668	1.12505	1028.89	0.571917	-4.5E-13	323.169	1.66909	176341.	220127.	2.56774	983.536	0.98148	114838.8	6.94
1.2	436.92	829.366	779.879	1.22732	1025.16	0.563272	0	315.309	1.77137	175091	217160.	2.55899	972.009	1.03439	115270.7	7.34
1.3	456.78	813.582	766.296	1.32960	1021.76	0.555459	3.41E-13	308.216	1.87365	173956.	214515	2.55108	961.425	1.08394	115675.8	7.72
1.4	476.64	799.114	753.782	1.43188	1018.64	0.548358	-1.1E-13	301.781	1.97593	172923.	212141.	2.54392	951.671	1.13046	116056.3	8.07
1.5	496.5	785.803	742.216	1.53416	1015.77	0.54187	1.14E-13	295.913	2.07820	171979.	209999.	2.53738	942.653	1.17422	116414.3	8.41
1.6	516.36	773.515	731.497	1.63643	1013.12	0.535917	2.27E-13	290.541	2.18048	171114.	208057.	2.53140	934.288	1.21548	116751.7	8.72
1.7	536.22	762.139	721.537	1.73871	1010.66	0.530432	-1.1E-13	285.603	2.28276	170317.	206287.	2.52590	926.507	1.25443	117070	9.02
1.8	556.08	751.574	712.257	1.84099	1008.39	0.52536	1.14E-13	281.047	2.38503	169582.	204669.	2.52083	919.251	1.29129	117370.8	9.30
1.9	575.94	741.739	703.592	1.94327	1006.26	0.520655	1.14E-13	276.831	2.48731	168901.	203183.	2.51614	912.467	1.32621	117655.4	9.56
2.0	595.8	732.558	695.483	2.04554	1004.28	0.516277	2.27E-13	272.917	2.58959	168269.	201813.	2.51179	906.110	1.35934	117925.1	9.81
							-1.4E-08									

One thing to note from the Worksheet of the previous page is the column labeled Solver. This is the column containing the momentum equation to be solved. The equation to be solved is in the form of equation (34) from Chapter 1. The Newton solver that comes with Microsoft EXCEL 5.0 was used to solve for a solution. One thing of note is that seeded properly (initial guess value), the solver can converge to either the first or second solution. In order to get a solution, and initial seed guess must be introduced into the solver, and it is from this seed that it progresses to use the Newton solver. So, in the process of solving the problem, careful attention was paid to which seed value was used. Because, for example, and initial guess of a supersonic end of mixing Mach number would preclude the solver from being able to converge to a subsonic root, and only Second solution answers would be obtained.

Another item of interest is the first row of mass flow ratio. The 0.0 mass flow ratio is simply the situation where the ejector is closed. This is exactly the same as the primary core acting independently. However, as a consequence of this spreadsheet being set up to handle multiple values of μ and P_r, different values for M_6 will show up as the solution to the $\mu = 0.0$ case, for various values of P_r. But this is not important, since the mixing plane for the zero mass flow ratio case is actually non-existent. The "expansion" that it undergoes is a "false" one, and is simply a consequence of the way the table is set up. The important thing to note is that the value of M_7 is the same in all cases, and thus the value of primary core thrust remains unchanged while varying P_r.

2.2: FIRST VERSUS SECOND SOLUTION COMPARISON

When looking at the solution to a supersonic cruise ejector problem, it has been noted that there are two solutions of interest. They are the subsonic mixing solution and the supersonic mixing solution. In order to analyze this problem completely, both solutions

must be compared. They will be compared by looking at entropy change and percent thrust increase.

2.2.1: First Solution Data

Since every entropy change calculated would be excessively cumbersome to present, and not entirely informative, selected values will be compared from each solution. The entropy changes that will be looked at are those for mass flow ratios 0, 1.0 and 2.0 for values of P_r = 0.9, 0.7, 0.55 and 0.4. The values of P_r correspond to bypass Mach numbers of 0.39, 0.73, 0.97 and 1.22 respectively. The above choices of points to evaluate are, of course, completely arbitrary, but give 12 points of comparison from a possible 240 (the number of data points in each thrust table used to create the forthcoming 3D graphs).

With regard to the percent thrust increase, all 240 points will be shown in both the subsonic and the supersonic case.

The values for the entropy change, for the first solution to the mixing problem at the previously indicated points are:

Table 4: First Solution Non-dimensional Entropy Changes

Values of $\frac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$3.28E^{-11}$	$-2.8E^{-16}$	$5.55E^{-16}$	$-5.6E^{-16}$
$\mu = 1.0$	0.924851	0.856494	0.862277	0.947219
$\mu = 2.0$	1.359347	1.281898	1.216471	1.361691

The values of percent thrust increase were all calculated by referencing the $\mu = 0.0$, $P_r = 0.9003$ case. For the first solution, the values of percent increase in thrust are shown in the following table and graphically displayed in the subsequent figure.

Table 5: First Solution Thrust Augmentation

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN

μ/ P_r=	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.5328	0.50	0.45	0.40
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.22**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.88	0.97	1.00	0.97	0.88	0.72	0.48	0.14	0.00	-0.31	-0.90	-1.67
0.2	1.71	1.92	2.01	2.04	2.00	1.90	1.70	1.40	1.27	0.98	0.40	-0.37
0.3	2.47	2.78	2.92	2.99	2.99	2.93	2.77	2.51	2.39	2.11	1.54	0.77
0.4	3.19	3.56	3.75	3.84	3.88	3.84	3.72	3.49	3.37	3.11	2.56	1.78
0.5	3.85	4.28	4.50	4.62	4.66	4.65	4.56	4.36	4.26	4.00	3.46	2.69
0.6	4.46	4.94	5.19	5.32	5.37	5.38	5.32	5.14	5.04	4.81	4.28	3.50
0.7	5.03	5.55	5.81	5.96	6.05	6.18	5.99	5.84	5.76	5.53	5.01	4.24
0.8	5.56	6.11	6.39	6.54	6.71	6.91	6.77	6.48	6.40	6.19	5.68	4.90
0.9	6.05	6.63	6.93	7.09	7.31	7.58	7.49	7.06	6.98	6.78	6.29	5.51
1.0	6.51	7.11	7.42	7.59	7.85	8.20	8.15	7.74	7.52	7.33	6.85	6.07
1.1	6.94	7.56	7.88	8.05	8.35	8.76	8.76	8.38	8.16	7.83	7.36	6.58
1.2	7.34	7.98	8.31	8.48	8.80	9.29	9.33	8.97	8.75	8.28	7.83	7.04
1.3	7.72	8.37	8.71	8.89	9.21	9.77	9.85	9.52	9.30	8.77	8.26	7.48
1.4	8.07	8.74	9.09	9.27	9.59	10.21	10.34	10.03	9.82	9.28	8.66	7.88
1.5	8.41	9.08	9.44	9.62	9.93	10.63	10.80	10.51	10.30	9.76	9.04	8.25
1.6	8.72	9.41	9.77	9.95	10.25	11.01	11.23	10.96	10.75	10.21	9.39	8.60
1.7	9.02	9.71	10.08	10.26	10.54	11.37	11.63	11.38	11.18	10.64	9.71	8.92
1.8	9.30	10.00	10.37	10.56	10.81	11.70	12.00	11.77	11.58	11.04	10.01	9.22
1.9	9.56	10.27	10.65	10.84	11.05	12.02	12.36	12.15	11.95	11.41	10.29	9.50
2.0	9.81	10.53	10.91	11.10	11.28	12.31	12.69	12.50	12.31	11.76	10.56	9.77

Two things to note about the previous table are, first, the boxed values indicate that the momentum equation could either not be solved, or choked flow occurred. To determine if choking resulted or if no solution occurred was a matter of looking at the column labeled "Solver" as was shown in Table 3. If the end of mixing Mach number was calculated to be 1 or close to it, and the value of the solver was zero, then only one solution was found (the choked solution). But, for all of the cases ran, that combination of factors rarely occurred for a Mach number exactly equal to 1 (However the end of mixing Mach number converged to 1 even if the solver was non-zero).

So, for all of the boxed values above, these values of thrust augmentation are not possible, since the momentum equation could not be solved for a given Mach number in the mixing plane. Thus, any value in a table of thrust augmentation boxed with a single line, is not considered a viable point where thrust augmentation could occur.

The second thing to note about the previous table is, the values of mass flow ratio are on the vertical axis of the graph and the horizontal axis has pressure recovery P_r, with bypass Mach number underneath.

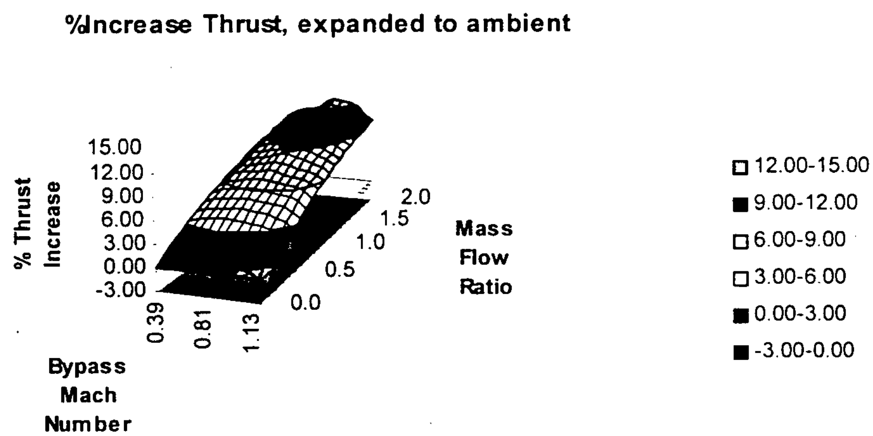

Figure 2: First Solution Graph of % Thrust Augmentation

2.2.2: Second Solution Data

For the second solution, there are some noticeable changes from the first solution. First of all, it is possible to violate the Second Law of Thermodynamics, and secondly, the thrust augmentation achieved is much larger, for similar conditions.

The entropy values corresponding to the same points as was presented in the first solution are

Table 6: Second Solution Non-dimensional Entropy Changes

Values of $\frac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$4.44E^{-16}$	$-2.8E^{-16}$	$5.55E^{-16}$	$-5.6E^{-16}$
$\mu = 1.0$	0.301295	0.856494	0.862277	0.904364
$\mu = 2.0$	-0.64536	1.281898	1.216471	1.341958

The table of thrust augmentation for the second solution shows a definite trend towards larger thrust increases. Yet also note that there are double outlined boxes which indicate the solution would violate the Second Law, and as before the single outline indicates solutions which do not solve the momentum equation. The corresponding table and figure for the second solution follow:

Table 7: Second Solution Thrust Augmentation

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN

μ/ P_r=	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.5328	0.50	0.45	0.40
	0.39	0.49	0.57	0.65	0.73	0.81	0.89	0.97	1.0000	1.05	1.13	1.22
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	1.26	1.15	1.14	1.15	1.15	1.14	1.13	1.11	1.10	1.09	1.05	1.01
0.2	2.69	2.27	2.18	2.17	2.17	2.16	2.14	2.11	2.10	2.06	2.00	1.93
0.3	4.28	3.35	3.13	3.09	3.09	3.08	3.06	3.01	2.99	2.95	2.87	2.76
0.4	5.98	4.41	4.01	3.93	3.93	3.92	3.89	3.84	3.81	3.76	3.65	3.51
0.5	7.79	5.45	4.83	4.69	4.68	4.68	4.65	4.59	4.56	4.49	4.37	4.21
0.6	9.70	6.49	5.61	5.39	5.38	5.38	5.35	5.27	5.24	5.17	5.02	4.84
0.7	11.69	7.51	6.34	6.04	6.05	6.18	5.99	5.91	5.87	5.79	5.63	5.42
0.8	13.77	8.54	7.04	6.64	6.71	6.91	6.77	6.50	6.45	6.36	6.19	5.96
0.9	15.92	9.56	7.71	7.20	7.31	7.58	7.49	7.06	7.00	6.89	6.70	6.45
1.0	18.14	10.58	8.36	7.72	7.85	8.20	8.15	7.74	7.52	7.39	7.18	6.91
1.1	20.43	11.60	8.98	8.21	8.35	8.76	8.76	8.38	8.16	7.85	7.63	7.34
1.2	22.78	12.63	9.59	8.67	8.80	9.29	9.33	8.97	8.75	8.29	8.04	7.74
1.3	25.18	13.66	10.18	9.11	9.21	9.77	9.85	9.52	9.30	8.77	8.43	8.11
1.4	27.64	14.69	10.76	9.53	9.59	10.21	10.34	10.03	9.82	9.28	8.79	8.46
1.5	30.14	15.73	11.33	9.93	9.93	10.63	10.80	10.51	10.30	9.76	9.13	8.79
1.6	32.69	16.77	11.89	10.31	10.25	11.01	11.23	10.96	10.75	10.21	9.45	9.09
1.7	35.29	17.82	12.44	10.68	10.54	11.37	11.63	11.38	11.18	10.64	9.76	9.38
1.8	37.93	18.88	12.98	11.04	10.81	11.70	12.00	11.77	11.58	11.04	10.04	9.65
1.9	40.61	19.94	13.52	11.38	11.05	12.02	12.36	12.15	11.95	11.41	10.31	9.90
2.0	43.32	21.00	14.05	11.71	11.28	12.31	12.69	12.50	12.31	11.76	10.57	10.14

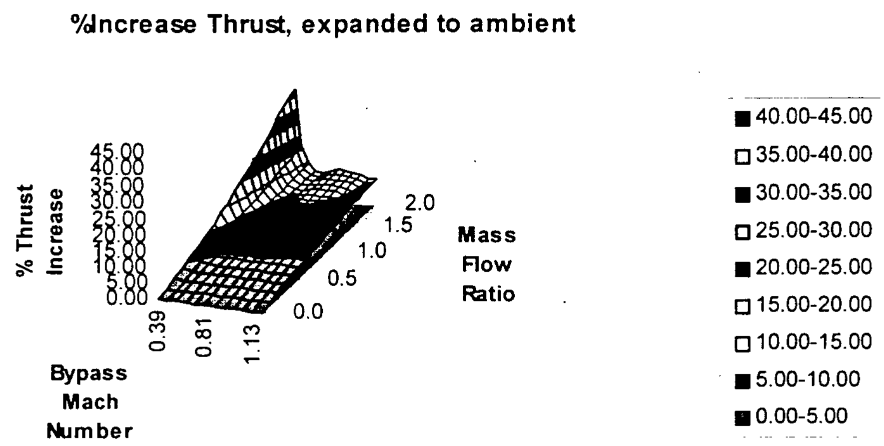

Figure 3: Second Solution Graph of % Thrust Augmentation

Looking at the graph above, one would be tempted to say that low bypass Mach numbers would be the best solution, due to the large spike in thrust augmentation. However, this would be a false conclusion, due to the violation of the Second Law that is indicated by the double outlined values in Table 7. Yet, the second solution still seems promising, since it has higher thrust augmentation then the first solution.

But, without an exact comparison of mixing lengths required, nothing can be said about which solution is better. Assuming a similar rate of mixing, the supersonic solution would take a longer distance to mix due to its greater flow velocity. Since nothing can be said about which solution provides more realizable thrust augmentation, without a valid mixing model, both results will be presented.

2.3: CONCLUSIONS

So, by looking at the first and second solution, one would want to say that the second

solution is unequivocally better, by virtue of thrust augmentation alone. However, no analysis was made of the actual distance required to mix, so the second solution's apparent benefits, could not be compared to the possible drawbacks of a longer mixing length. The thrust augmentation increase may be offset by the associated weight penalty of a longer mixing length. There is room for further analysis in this area, and if this is to be done a model for mixing must be made.

CHAPTER 3: DIFFUSER LOSSES

After looking at the simple case of the ejector in cruise configuration for a subsonic and supersonic mixing solution, it is natural to extend this by attempting to account for diffuser losses. It was assumed that the entrained ejector air was brought in through the diffuser without any loss in total pressure. For the case without losses, this was all right, but assuming the ejector diffuser is imperfect and may have some losses associated with it, losses must be introduced. This was done by simply taking a percentage loss in stagnation pressure from the ambient flow.

3.1: EFFECT OF DIFFUSER LOSS

This subsequent decrease in ejector total pressure, affects its static pressure too, which therefore affects the full expansion of the primary flow. Losses were taken in 5% increments from 5% to 25%. The affects of these losses are shown in the following tables and charts.

3.1.1: 5% DIFFUSER LOSS (1ST AND 2ND SOLUTION)

This section and the following sections will have information on both the first and the second solution. Tables of entropy changes, percent thrust increase may be found in Appendix C; however, the graphs of the thrust augmentation will be included in each section.

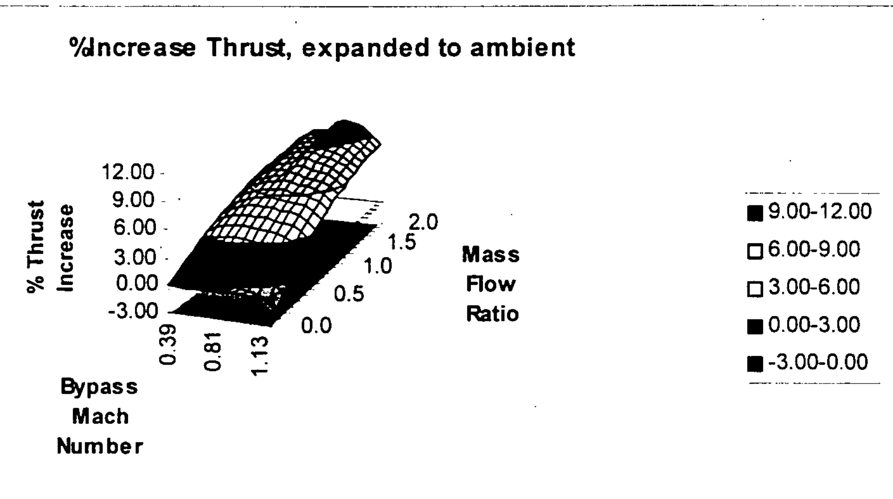

Figure 4: Graph of % Thrust, 5% Loss (1st Solution)

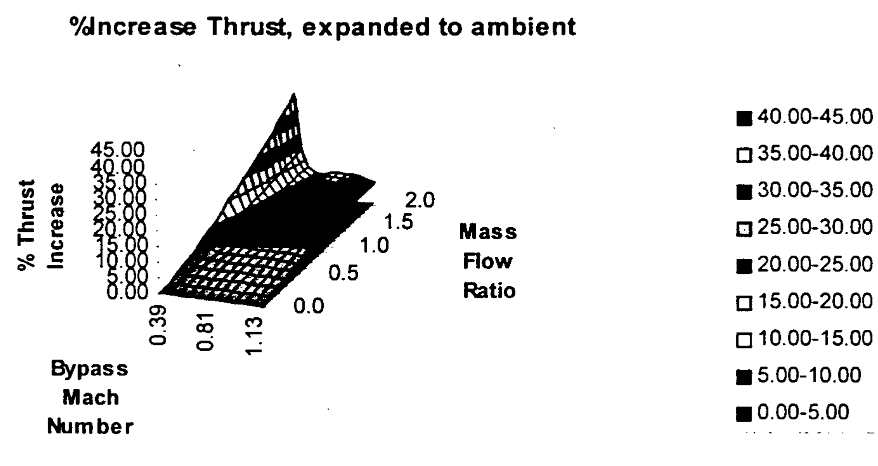

Figure 5: Graph of % Thrust, 5% Loss (2nd Solution)

3.1.2: 10% Diffuser Loss (1st and 2nd Solution)

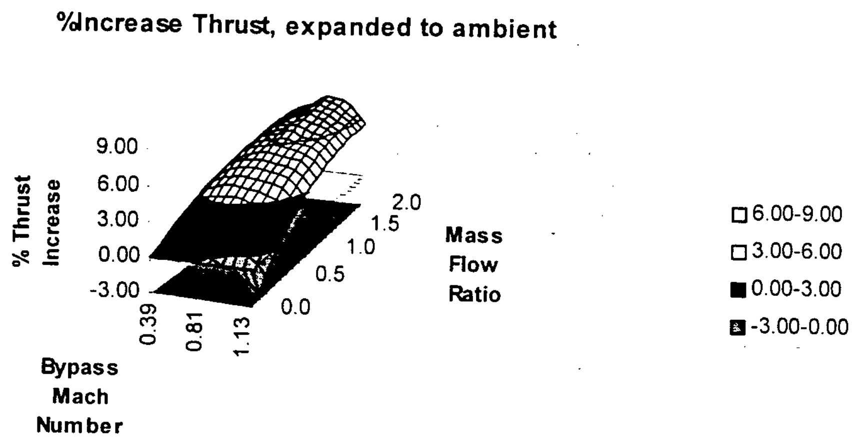

Figure 6: Graph of % Thrust, 10% Loss (1st Solution)

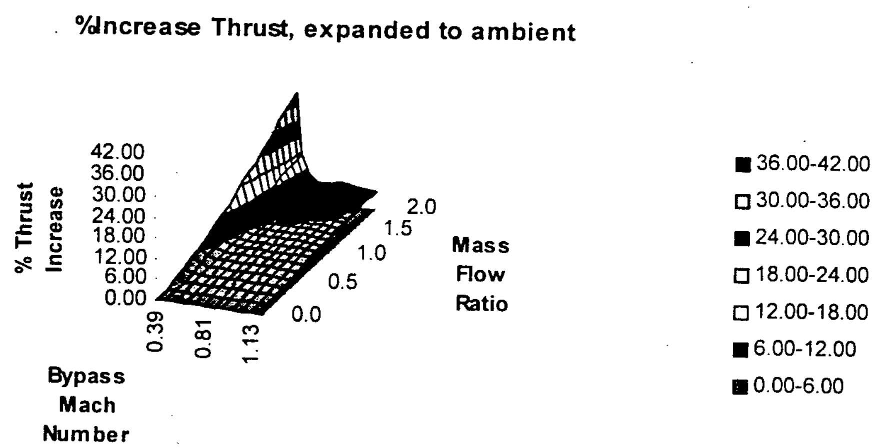

Figure 7: Graph of % Thrust, 10% Loss (2nd Solution)

3.1.3: 15% Diffuser Loss (1st and 2nd Solution)

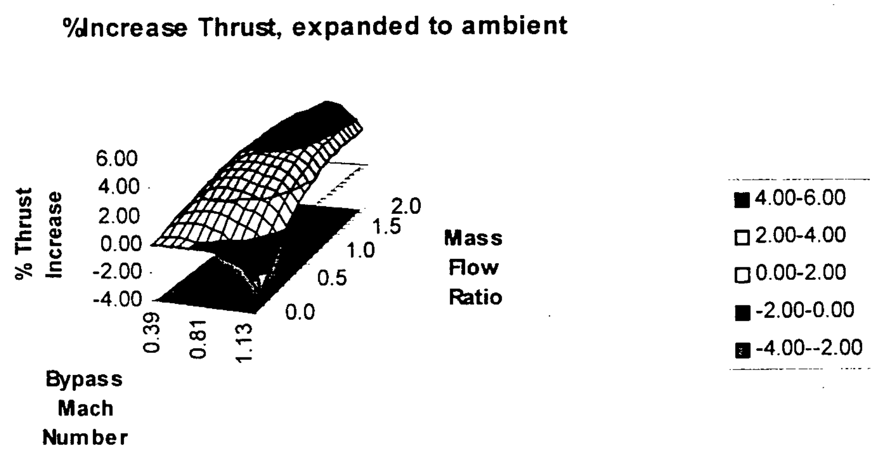

Figure 8: Graph of % Thrust, 15% Loss (1st Solution)

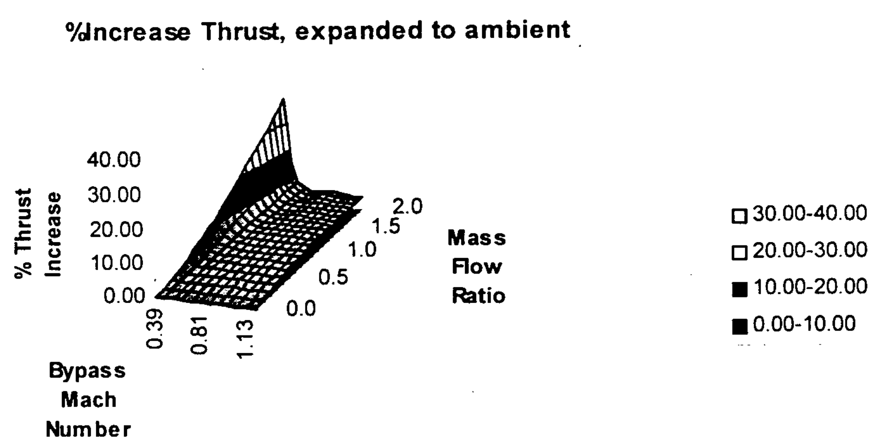

Figure 9: Graph of % Thrust, 15% Loss (2nd Solution)

3.1.4: 20% Diffuser Loss (1st and 2nd Solution)

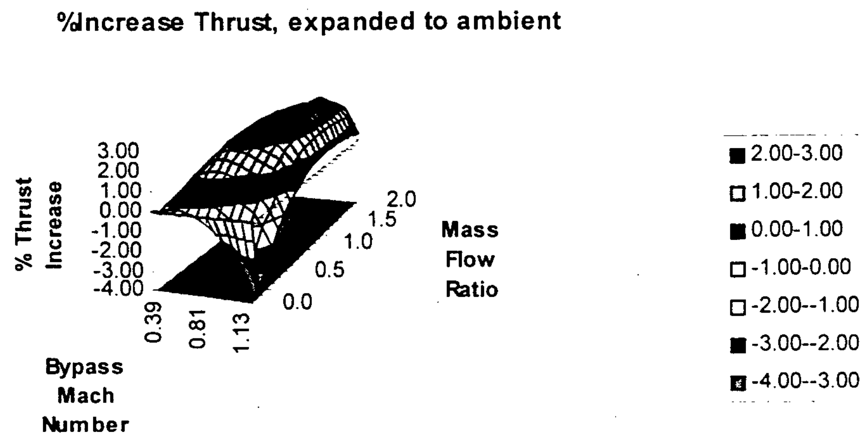

Figure 10: Graph of % Thrust, 20% Loss (1st Solution)

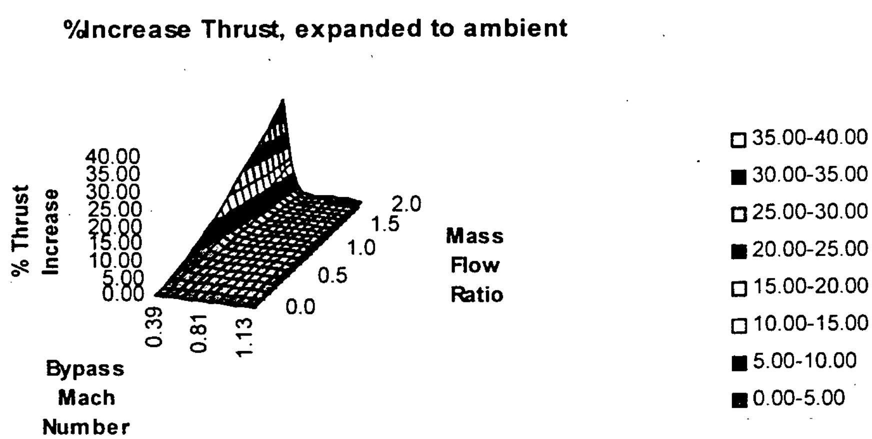

Figure 11: Graph of % Thrust, 20% Loss (2nd Solution)

3.1.5: 25% DIFFUSER LOSS (1ST AND 2ND SOLUTION)

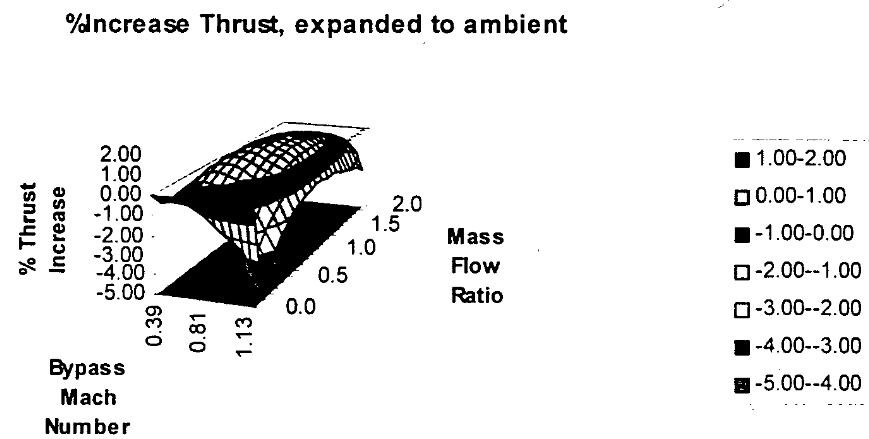

Figure 12: Graph of % Thrust, 25% Loss (1st Solution)

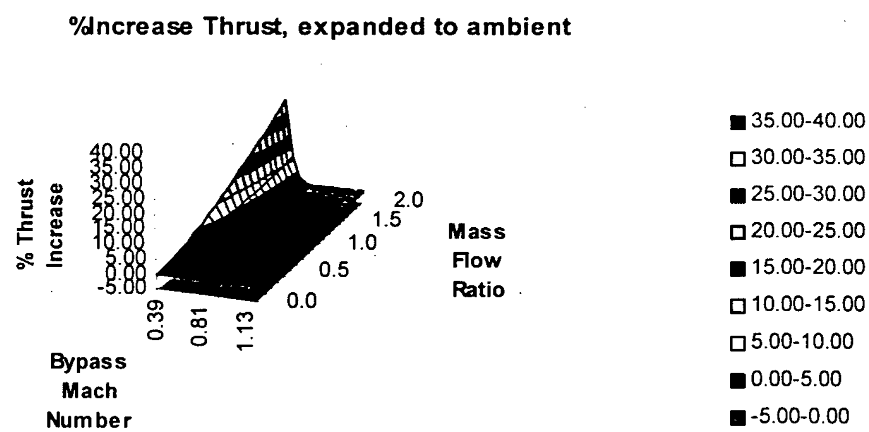

Figure 13: Graph of % Thrust, 25% Loss (2nd Solution)

3.2: COMMENTS ON DIFFUSER LOSS

Looking at the previous subsections one can see an overriding trend of thrust augmentation decrease. As the diffuser becomes less efficient at recovering total pressure from the ambient flow, the thrust of the ejector combination begins to decline.

Once again the second solution has a larger thrust augmentation, but once again certain parts are defeated by the Second Law of Thermodynamics. While nothing can be said about which solution is easier to achieve practically (the first or the second), they both show a positive non-dimensional entropy change over a large range of values. So, both solutions are of interest, and should be examined.

Also, the tables of thrust augmentation still show a problem with the momentum equation not being satisfied. However, one interesting trend is the disappearance of this phenomena with increased diffuser loss. The flow becomes closer to being choked (or sonic), and this can be explained physically, by the following. The choking occurs, since the primary stream is trying to expand to the secondary stream static pressure. In doing so, there are certain configurations of mass flow ratio and bypass Mach number (the single boxed values) where the primary flow continues to expand in the ejector, but leads to an effective constriction of the area the secondary flow must pass through. This restricted area acts as a virtual throat for the secondary flow to pass through. Regardless of whether the bypass (secondary) flow is initially subsonic or supersonic, the restriction in area acts to bring the flow back to sonic. A similar process is simultaneously happening with the primary flow, so it goes sonic too. With increasing losses in secondary stream stagnation pressure, there is also an associated loss in static pressure, since the bypass Mach number remains the same. This more readily allows for a solution of the momentum equation that is not choked.

CHAPTER 4: BLEED LOSSES

4.1: TREATMENT OF BLEED LOSS

After doing the analysis given so far, the results were presented to the Boeing Company. During the meeting with Boeing, Mr. Jan Syberg noted that there was no account for bleed losses. This led the author to consider bleed losses in his analysis, and to model something similar to what Boeing said was feasible, a six percent diffuser loss along with a five percent bleed from the ejector was what was used for the model.

An attempt to model bleed loss was done by treating it as a throttling process. It was assumed that the analysis done previously would be accurate for the flow that was not bled from ejector's diffuser. So, since the previous analysis valid, the bleed flow could be handled separately. The bleed flow was calculated as

$$\dot{m}_b = \left(\frac{\dot{m}_5}{0.95} - \dot{m}_5 \right) = \dot{m}_5\,(0.05263) \qquad (48)$$

This quantity was fixed by continuity, but for the process of throttling, the bleed flow was considered to be adiabatic, isentropically expanded and then shocked back to a lower pressure. For one case it was shocked back to atmospheric pressure, and for the second case, it was throttled to lose one-half of its stagnation pressure.

The figure of the next page shows how the process for the first case was modeled:

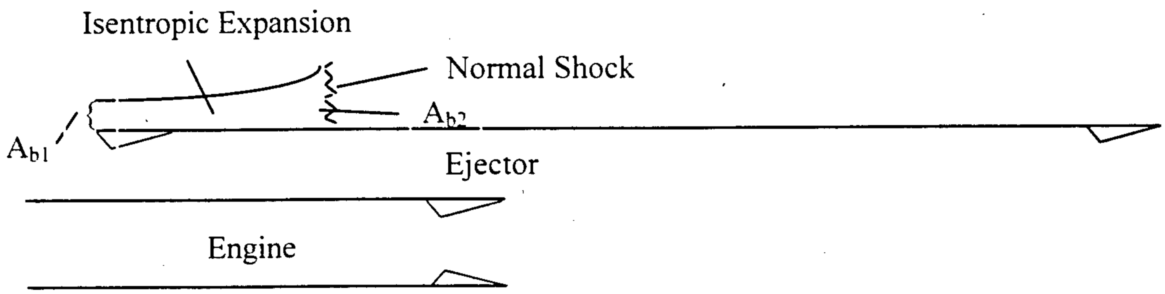

Figure 14: Throttling Process Diagram for Throttle to Ambient

The value for A_{b1} was calculated by the following equation.

$$A_{b1} = \frac{\dot{m}_b}{\rho_b v_b} = \frac{\dot{m}_b}{\rho_1 v_1} \tag{49}$$

This assumes that the inlet velocity for the bleed flow is exactly the same as the translational velocity of the aircraft.

For the second case, an attempt was made to bleed more intelligently. This led to the result of losing 1/2 P_0. To do this the bleed flow was bled just prior to the point in the diffuser where the flow was sonic. i.e. $M = 1$. From there it would be shocked and then expanded to ambient pressure. Across this shock, from the Mach 2.4 flow, the ratio of stagnation pressure after the shock (P_{02}), to that before the shock (P_{01}) is $P_{02}/P_{01} \approx 0.5$.[7] So the choice was made to use 1/2 P_0 as the model for bleed in the second case. A diagram of the process of the second case is the following:

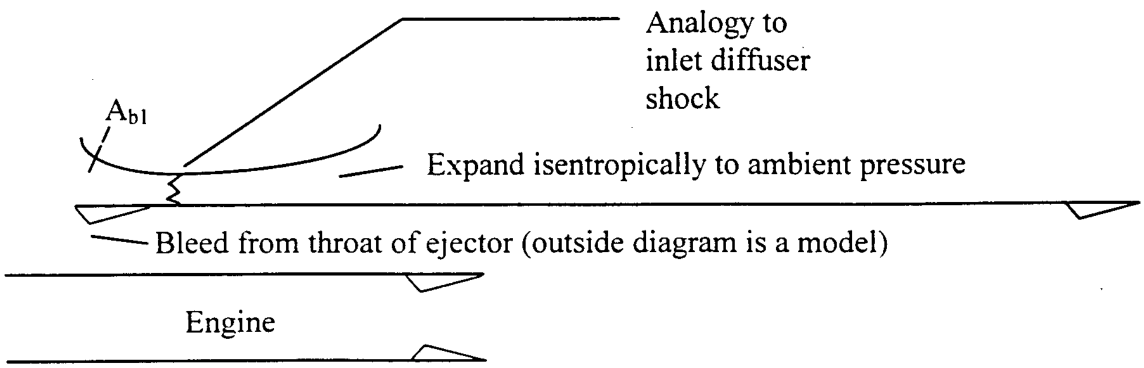

Figure 15: Throttling Process Diagram for
Throttle to 1/2 P_0

For purposes of the following derivations both equations (48) and (49), that were given previously, hold.

4.1.1: THROTTLE TO ATMOSPHERIC PRESSURE

For the first case, it was desired that the pressure after the shock would equal atmospheric pressure. So, to do this the following relations and variable definitions were used. P_0 is the stagnation pressure of the flow, P_e is the static pressure at the exit of the throttle prior to shock, and P_2 is the static pressure of the flow after the shock.

$$P_e = P_o\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{-\gamma}{\gamma-1}}$$

$$M_2^2 = \frac{M_e^2 + \frac{2}{\gamma - 1}}{2 M_e^2 \frac{\gamma}{\gamma - 1} - 1}$$

$$\frac{P_2}{P_e} = \frac{1 + \gamma M_e^2}{1 + \gamma M_2^2}$$

Combining these gives:

$$\frac{P_2}{P_o \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{-\gamma}{\gamma - 1}}} = \frac{1 + \gamma M_e^2}{1 + \gamma \left(\frac{M_e^2 + \frac{2}{\gamma - 1}}{2 M_e^2 \frac{\gamma}{\gamma - 1} - 1}\right)} \tag{50}$$

This holds for any throttling process. However, for the first case, the static pressure after the shock was desired to be atmospheric pressure. This is to say that $P_2 = P_1$. Another relation that will simplify the equation is that in air (where $\gamma = 1.4$), $P_0 = {}^{1}/_{0.0684} * P_1$. (Cite the Emmons Gas dynamics tables) Substituting these back into (50) and performing some algebra gives:

$$\frac{P_2}{\frac{1}{0.0684} P_2 \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{-\gamma}{\gamma - 1}}} = \frac{\left(1 + \gamma M_e^2\right)\left(2 M_e^2 \frac{\gamma}{\gamma - 1} - 1\right)}{2 M_e^2 \frac{\gamma}{\gamma - 1} - 1 + \gamma \left(M_e^2 + \frac{2}{\gamma - 1}\right)}$$

Canceling the P_2, noting that for air, ${}^{\gamma-1}/_2 = .2$, ${}^{\gamma}/_{\gamma-1} = 3.5$, and ${}^{2}/_{\gamma-1} = 5$; and finally replacing M_e with M.

$$\frac{0.0684}{\left(1+0.2M^2\right)^{-3.5}} = \frac{\left(1+1.4M^2\right)\left(7M^2-1\right)}{7M^2-1+1.4\ \left(M^2+5\right)}$$

Solving numerically for M gives $M = 4.714162 \approx 4.7$. Now that the Mach number to which the flow must be isentropically expanded is known, the thrust decrease may be calculated from the following formulas.

$$F_{loss} = F_{ML} + F_{PL}$$

$$F_{ML} = \dot{m}_b\ \left(v_2 - v_1\right)$$

$$F_{PL} = (P_2\text{-}P_1)\ A_{b2}$$

where F_{ML} is the thrust loss due to momentum decrease and F_{PL} is the thrust "loss" due to pressure differences.

For this first case, since $P_2 = P_1$, the thrust "loss" due to pressure is zero, and this only leaves the momentum loss term. To calculate this term, the value of v_2 must be calculated. To calculate this velocity, one must note that the speed of sound used to calculate v_2 is different from that of the one used to calculate v_1. This is due to the temperature change that occurred in the flow while it was being isentropically and adiabatically expanded to $M = 4.7$.

Even though A_{b2} will not be used in this first case, the process of its calculation will be covered, since it will actually be used in the second case. The normal shock relations for $M = 4.7$ that will be used to calculate A_{b2} and v_2 are as follows

$A_{4.7}/A^* = 19.58$ @ $M = 4.7$

$A_{2.4}/A^* = 2.4031$ @ $M = 2.4$

$T/T_0 = 0.46468$ @ $M = 2.4$

$T/T_0 = 0.18457$ @ $M = 4.7$

$T_2/T_e = 5.2334$ (across a M= 4.7 normal shock)

Using these to calculate A_{b2} gives

$$A_{b2} = \frac{A_{4.7}}{A^*}\frac{A^*}{A_{2.4}}A_{b1} = \frac{19.58}{1}\frac{1}{2.4031}A_{b1} = 8.2602\ A_{b1}$$

Now to calculate v_2, the new static temperature must be calculated. This can then be substituted into the following equation to give v_2. This equation can also be used to calculate v_1.

$$v = M\sqrt{\gamma\ R\ T}$$

For the free stream velocity one gets the following

$$v_1 = 2.4\sqrt{1.4\cdot(287\ \mathrm{J/kg\cdot K})\cdot(216.7\ \mathrm{K})} = 708.183\ \mathrm{m/s}$$

Using the shock relations detailed above, the static temperature after the shock is

$$T_2 = 5.2334\ T_e = 5.2334 \cdot T_1 \cdot T_{01}/T_1 \cdot T_e/T_{oe} = 5.2334 \cdot 216.7 \cdot {}^1/_{0.46468} \cdot {}^{0.18457}/_1 = 450.4535\ \mathrm{K}$$

So in this new flow $M = 0.41992$, via the normal shock relation for $M = 4.7$, and using the definition of velocity above one gets

$$v_2 = 0.41992\sqrt{1.4\cdot(287)\cdot(450.4535)} = 178.647\ \mathrm{m/s}$$

This then gives momentum thrust loss as

$$F_{ML} = \dot{m}_b\ (v_2 - v_1) = \dot{m}_b\ (178.647 - 708.183) = \dot{m}_b\ (-529.5359\ \mathrm{m/s})$$

Now with these new values calculated, five new columns were added to the previously used spreadsheets. These columns were $\dot{m}_b$, A_{b1}, A_{b2}, F_{ML} and F_{PL}. Adding the values of momentum thrust and pressure thrust into the previously calculated trust gives new values for overall thrust and leads to a change in the percent thrust increase.

4.1.2: THROTTLE TO LOSE HALF STAGNATION PRESSURE

Due to the reasons stated before, the throttle to 1/2 P_0 was chosen as the second case. But, in order to evaluate this case, a slight variation on the equations just derived must be calculated. While equation (50) above would hold for this case, it is not in a convenient form to relate stagnation pressures. Therefore the following derivation was used.

$$\frac{P_{20}}{P_{10}} = \frac{P_{20}}{P_1}\frac{P_1}{P_{10}} \quad \text{where} \quad \frac{P_{20}}{P_1} = \frac{P_{20}}{P_2}\frac{P_2}{P_1}$$

$$\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2} \quad \text{and} \quad \frac{P_{0i}}{P_i} = \left(1+\frac{\gamma-1}{2}M_i^2\right)^{\frac{\gamma}{\gamma-1}}$$

Since we want P_{20} to be $^1/_2\, P_{10}$, then

$$\frac{1}{2} = \frac{P_{20}}{P_{10}} = \frac{P_{20}}{P_2}\frac{P_1}{P_{10}}\frac{P_2}{P_1} = \frac{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}}\frac{1+\gamma M_1^2}{1+\gamma M_2^2}$$

since $M_2^2 = \dfrac{M_1^2 + \dfrac{2}{\gamma-1}}{2M_1^2\dfrac{\gamma}{\gamma-1} - 1}$, substitution into the above with $\gamma = 1.4$ gives

$$\frac{1}{2}=\frac{\left(1+0.2\dfrac{M_1^2+5}{7M_1^2-1}\right)^{3.5}}{\left(1+0.2M_1^2\right)^{3.5}}\frac{1+1.4\ M_1^2}{1+1.4\dfrac{M_1^2+5}{7M_1^2-1}}$$

After performing some algebra, and letting $M_1 = M$, one gets

$$\frac{1}{2}=\left(\frac{1+\dfrac{1+0.2M^2}{7M^2-1}}{1+0.2M^2}\right)^{3.5}\frac{\left(7M^2-1\right)\left(1+1.4\ M^2\right)}{7M^2-1+1.4\left(M^2+5\right)}$$

$$\frac{1}{2}=\left(\left(1+0.2M^2\right)^{-1}+\left(7M^2-1\right)^{-1}\right)^{3.5}\frac{9.8M^4+5.6M^2-1}{8.4M^2+6}$$

Numerically solving for M, one finds that an exit Mach number of $M = 2.4975 \approx 2.5$ prior to the shock gives a decrease in stagnation pressure of $^1/_2$. Thus, the throttling process involves an expansion from $M = 2.4$ to $M = 2.5$.

Expanding the flow to Mach 2.5 creates a new area ratio as well as a new static temperature and therefore speed of sound. The important information from the normal shock tables that will be needed to calculate A_{b2} and T_2 are

$A_{2.5}/A^* = 2.6367$ @ $M = 2.5$

$T/T_0 = 0.44444$ @ $M = 2.5$

$T_2/T_e = 2.1375$ (across a $M = 2.5$ normal shock)

using these and the information from the previous section.

$$A_{b2}=\frac{A_{2.5}}{A^*}\frac{A^*}{A_{2.4}}A_{b1}=\frac{2.6367}{1}\frac{1}{2.4031}A_{b1}=1.0972\ A_{b1}$$

$T_2 = 2.1375\ T_e = 2.1375 \cdot T_1 \cdot T_{01}/T_1 \cdot T_e/T_{oe} = 2.1375 \cdot 216.7 \cdot {}^1/_{0.46468} \cdot {}^{0.44444}/_1 = 443.0209$ K

However, as before there is a decrease in Mach number across a normal shock and for $M = 2.5$, the Mach number decreases to $M = 0.51299$ and plugging these in gives

$$v_2 = 0.51299\sqrt{1.4 \cdot (287) \cdot (443.0209)} = 207.261 \text{ m/s}$$

This then gives momentum thrust loss as

$$F_{ML} = \dot{m}_b\ (v_2 - v_1) = \dot{m}_b\ (207.261 - 708.183) = \dot{m}_b\ (\text{-}491.749 \text{ m/s})$$

But, one more piece of information is needed to calculate F_{PL}. The pressure on the other side of the shock is needed. This can be calculated in terms of the initial inlet static pressure since stagnation pressure is conserved up to the point of the shock, and then there is a relation between the static pressures before and after the shock. For the isentropic expansion to $M_e = 2.5$

$$P_e = P_o\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{-\gamma}{\gamma-1}} = P_o\ (0.05853) = \frac{P_1\ (0.05853)}{0.0684} = 0.8557\ P_1$$

but across a Mach 2.5 normal shock

$P_2/P_e = 7.125$ so $P_2 = 7.125\ P_e = 6.0969\ P_1$

now, substituting all of these into the equation for pressure thrust "loss" gives

$F_{PL} = (P_2 - P_1)\ A_{b2} = (6.0969\ P_1 - P_1)\ 1.0972\ A_{b1} = 5.5923\ P_1\ A_{b1}$

which is actually a thrust gain. It is a "gain" in thrust, since the direction of the net force due to the pressure difference acts in the direction of flight and acts to propel the vehicle.

Yet this does not lead to something for nothing, since when the momentum thrust loss is also accounted for, there is a net decrease in overall thrust. This will be shown in the following section, where the two throttling processes will be compared.

4.2: THROTTLING RESULTS AND COMPARISON

In the previous chapter, it was discovered that the second solution to the ejector problem provides larger thrust augmentations, but there were cases where the entropy changes were negative. So, to simplify presenting the trends of the results, the second solution will not be covered in the following comparisons. Also, the entropy changes will not be given, because the entropy changes that occur from a 6% diffuser loss do not differ greatly from the 5% loss discussed before.

After calculating the thrust decrease due to throttling the flow back to ambient pressure, the following result was achieved.

Table 8: Thrust Augmentation with 6% Diffuser Loss and Bleed Flow to Ambient Pressure

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN, Diffuser loss=6%

μ/ P_r=	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.5328	0.50	0.45	0.40
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.20**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.12	0.23	0.25	0.20	0.09	-0.10	-0.37	-0.74	-0.89	-1.22	-1.86	-2.67
0.2	0.26	0.51	0.62	0.64	0.59	0.46	0.24	-0.09	-0.24	-0.55	-1.17	-1.98
0.3	0.36	0.71	0.89	0.96	0.96	0.88	0.69	0.40	0.27	-0.04	-0.64	-1.46
0.4	0.41	0.85	1.07	1.19	1.22	1.17	1.03	0.77	0.64	0.35	-0.24	-1.05
0.5	0.42	0.92	1.19	1.33	1.39	1.37	1.26	1.03	0.91	0.64	0.06	-0.76
0.6	0.39	0.95	1.24	1.41	1.48	1.49	1.41	1.20	1.09	0.83	0.26	-0.56
0.7	0.32	0.92	1.24	1.42	1.51	1.53	1.47	1.29	1.19	0.95	0.39	-0.43
0.8	0.21	0.85	1.19	1.39	1.49	1.52	1.48	1.32	1.23	0.99	0.45	-0.38
0.9	0.07	0.74	1.10	1.31	1.41	1.57	1.46	1.29	1.21	0.98	0.45	-0.38
1.0	-0.10	0.60	0.98	1.19	1.29	1.57	1.51	1.21	1.13	0.92	0.39	-0.44
1.1	-0.29	0.42	0.81	1.03	1.14	1.52	1.51	1.10	1.01	0.81	0.30	-0.54
1.2	-0.52	0.22	0.62	0.85	0.95	1.43	1.46	1.08	0.86	0.66	0.16	-0.68
1.3	-0.76	-0.01	0.40	0.63	0.74	1.30	1.37	1.02	0.80	0.48	-0.02	-0.85
1.4	-1.03	-0.26	0.16	0.39	0.50	1.13	1.25	0.92	0.70	0.26	-0.22	-1.06
1.5	-1.32	-0.54	-0.11	0.13	0.23	0.93	1.09	0.78	0.57	0.01	-0.46	-1.30
1.6	-1.62	-0.83	-0.39	-0.15	-0.06	0.70	0.91	0.62	0.41	-0.15	-0.72	-1.56
1.7	-1.94	-1.14	-0.70	-0.46	-0.36	0.45	0.70	0.43	0.23	-0.33	-1.00	-1.85

Table 8: Continued

1.8	-2.28	-1.47	-1.02	-0.78	-0.68	0.17	0.46	0.22	0.01	-0.54	-1.30	-2.15
1.9	-2.63	-1.81	-1.36	-1.11	-1.02	-0.13	0.20	-0.02	-0.22	-0.78	-1.63	-2.48
2.0	-3.00	-2.17	-1.71	-1.46	-1.37	-0.45	-0.07	-0.28	-0.47	-1.04	-1.97	-2.82

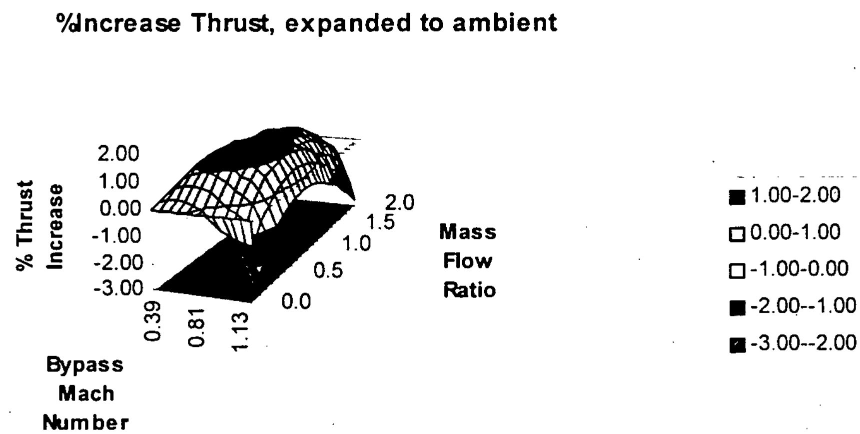

Figure 16: Graph of % Thrust Increase with
Bleed to Ambient Pressure

The result of throttling the flow to half of its previous stagnation pressure results in the following thrust increases.

Table 9: Thrust Augmentation with 6% Diffuser
Loss and Bleed Flow to $^1/_2\ P_0$

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN, Diffuser loss=6%

μ / P_r=	**0.90**	**0.85**	**0.80**	**0.75**	**0.70**	**0.65**	**0.60**	**0.55**	**0.5328**	**0.50**	**0.45**	**0.40**
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.20**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.64	0.75	0.77	0.72	0.60	0.42	0.15	-0.22	-0.38	-0.71	-1.34	-2.15
0.2	1.29	1.54	1.65	1.67	1.62	1.49	1.27	0.94	0.79	0.48	-0.14	-0.96
0.3	1.90	2.26	2.43	2.51	2.50	2.42	2.24	1.94	1.81	1.51	0.90	0.09
0.4	2.47	2.91	3.13	3.25	3.28	3.23	3.09	2.82	2.70	2.41	1.82	1.01
0.5	3.00	3.50	3.76	3.90	3.97	3.95	3.84	3.60	3.49	3.21	2.63	1.81
0.6	3.48	4.03	4.33	4.50	4.57	4.58	4.50	4.29	4.18	3.92	3.35	2.53
0.7	3.92	4.52	4.85	5.03	5.12	5.14	5.08	4.90	4.80	4.55	3.99	3.17

Table 9: Continued

0.8	4.33	4.97	5.31	5.51	5.60	5.64	5.60	5.44	5.35	5.11	4.57	3.74
0.9	4.71	5.37	5.74	5.94	6.04	6.21	6.09	5.92	5.84	5.62	5.08	4.25
1.0	5.05	5.75	6.12	6.34	6.44	6.72	6.66	6.35	6.28	6.07	5.54	4.71
1.1	5.37	6.09	6.48	6.70	6.80	7.19	7.17	6.76	6.67	6.47	5.96	5.13
1.2	5.66	6.40	6.80	7.03	7.13	7.61	7.64	7.26	7.03	6.84	6.34	5.50
1.3	5.93	6.68	7.10	7.33	7.43	7.99	8.06	7.71	7.49	7.17	6.68	5.84
1.4	6.18	6.95	7.37	7.60	7.71	8.34	8.46	8.12	7.91	7.47	6.99	6.15
1.5	6.41	7.19	7.62	7.85	7.95	8.65	8.82	8.51	8.29	7.74	7.27	6.43
1.6	6.62	7.41	7.84	8.08	8.18	8.94	9.15	8.86	8.65	8.09	7.52	6.68
1.7	6.81	7.61	8.05	8.30	8.39	9.20	9.45	9.18	8.98	8.42	7.75	6.91
1.8	6.99	7.80	8.25	8.49	8.58	9.43	9.73	9.48	9.28	8.72	7.96	7.11
1.9	7.15	7.97	8.42	8.67	8.76	9.65	9.99	9.76	9.56	9.00	8.15	7.30
2.0	7.30	8.13	8.59	8.83	8.92	9.84	10.22	10.02	9.82	9.26	8.33	7.47

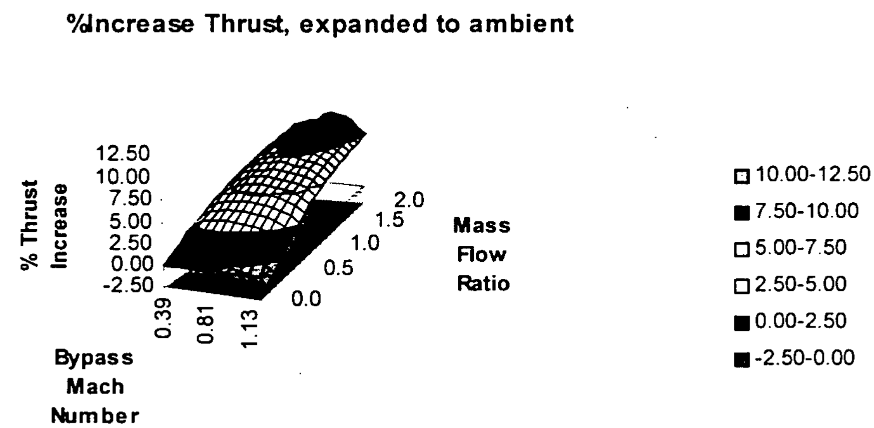

Figure 17: Graph of % Thrust Increase with Bleed to $^1/_2\ P_0$

Looking at the two previous graphs, it is apparent that the process of throttling to ambient pressure has a greater affect on decreasing realizable thrust. When the flow is only throttled to $^1/_2\ P_0$, there is a less drastic affect on overall thrust. This logically makes sense, since the stronger the shock that the flow has to pass through, the greater the decrease in the energy it has to provide thrust. The basic affect of the throttle to ambient

pressure is to act like a large drag brake. It almost completely removes any thrust gain from the ejector/engine combination, but it does not completely destroy the thrust gained.

Thus, as can be seen from the first case, even if the flow is treated in a very severe manner the thrust gained is not completely destroyed. Hopefully this would carry through into any actual bleed configuration used. The only way that the thrust could be completely removed would be to throttle to a pressure even lower then ambient, but this would be extremely irrational. Hence, unless something illogical is done to the flow, there will still be a thrust increase for a 6% diffuser loss. Larger diffuser losses were not examined, but the thrust decrease would be taken in a similar manner off of the already decreased thrust profiles of the larger diffuser loss cases.

CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

5.1: EJECTORS AS A VIABLE OPTION?

In the Boeing design for the HSCT, ejectors are going to be used for the purpose of noise damping. In order to determine if ejectors would provide thrust augmentation, in the supersonic cruise configuration, the analysis just presented was conducted.

5.1.1: A VIABLE FIRST ORDER ANALYSIS

The derivation of Chapter 1 (with results presented in Chapter 2) showed a considerable thrust augmentation. This gain occurred over a wide range of bypass Mach numbers and mass flow ratios, when no losses, other than mixing were taken into account.

Both the first and the second solutions show a large percentage increase over the baseline thrust. Looking at the first solution, there are some cases where the momentum equation could not be solved, and to reiterate, these values were boxed in to indicate that this was so. Yet, even if the boxed solutions are not possible, there is still a large thrust increase that can be gained.

As for the second solution, the thrust increases are larger, but whether this would be more beneficial cannot be determined from the analysis presented. A model for determining mixing length was not included, so no statement can be made regarding its possible benefits over the first solution.

One final thing to note regarding the second solution is that there are some values of bypass Mach number and mass flow ratio that do not allow for a solution because of the Second Law of Thermodynamics. The associated non-dimensional entropy changes are negative, since the second solution to the momentum equation is supersonic, but the

incoming flows are both subsonic. Therefore, this causes the negative entropy change, and is not physically possible.

5.1.2: Effect of Diffuser Loss on Thrust

When the affects of a straight percentage diffuser loss were applied to the stagnation pressure of the secondary flow, there was a noticeable decrease in the thrust augmentation. This occurred for both the first and the second solutions.

However, in both cases, there was not a total loss in thrust augmentation until the diffuser loss reached 25%. Therefore, under the assumptions of the analysis of this paper, ejectors still seem to have promise even with a diffuser loss added.

5.1.3: Effect of Bleed Loss on Thrust

Looking back at Chapter 4, the bleed loss was modeled in two way. First, the thrust was throttled directly to ambient pressure, and the second case had the bleed flow lose half of its stagnation pressure. The reasons for the second choice were previously, but by attempting to bleed the flow more "intelligently", the thrust losses experienced were not nearly as large.

For the first case, the losses were enough to make any thrust gains from the ejector practically disappear. This was mainly due to the strong normal shock that was present from doing the bleed in the manner proposed. The throttling model that was used created such an adverse affect on mass flow; the result was an essential drag brake.

But, doing the bleed so that there was only a loss in half the stagnation pressure, the results are more tolerable, and all of the thrust augmentation is not destroyed.

5.2: WHAT NEEDS TO BE DONE

After doing this analysis, the author determined ejectors seem very promising for further research. Some of the areas that can be looked into are, extending the mathematical modeling of the problem itself, and beginning experimental testing.

5.2.1: THE MATHEMATICAL MODEL

Some of the shortcomings of the model used in the analysis were, for example; all gases were assumed to be ideal, and the ratio of specific heats was constant. While these make good first order assumptions, the favorable results achieved thus far lead the author to conclude that the model should be refined and examined further.

One of the first adaptations to make is to longer assume that the gases that will be mixing are ideal. Real gas effects can be added into the model, and their effect on the analysis may then be determined.

Second, the gases were all assumed to be air, and this would not be so in the actual working model for the HSCT. There would be combustible gases expelled from the primary core (the engine) and these could possibly combust with the secondary (entrained air) flow. So, adding combustion to the model seems a reasonable extension.

Third, the author's analysis was essentially an analysis of the performance of an engine that was uninstalled. No external drag forces were accounted for, so if desired, one could examine in greater detail the effect of drag on the system over various possible inlet geometry's.

Fourth, there is no model being used to determine the exact method of mixing in the thrust (mixing) chamber. It was assumed that mixing was complete when the mixing Mach number (M_6) was determined. But, nothing was done to calculate the distance that would be required to mix. So, a very interesting area of research would be an

examination of how to make the two flows mix in as short a distance as possible. This would make the thrust augmentation an even greater benefit, since the weight of the ejector assembly used to generate thrust would decrease simply due to the shortened mixing length.

Finally, a different model for diffuser and bleed loss could be attempted. The exact manner in which it was done by the author, may not be the most elegant way to do so. Solving the exact characteristics of the inlet diffuser loss, and bleed loss, and how to minimize both is an area of significant importance to the success of this endeavor. If these two could be kept as low as possible, the thrust augmentation achieved is of even greater value.

5.2.2: Experimental Testing

Above and beyond the intricacies of theoretical modeling, some actual hardware analyses and experiments should be conducted. Empirical testing sometimes serves as a guide to the theoretician, and would be very helpful for this project. The exact dynamic that causes the solution of mixing to literally choke, and sometimes be impossible is not readily apparent from the math of theory alone. Examining this phenomena, and the thrust augmentations that appear from theory, would provide valuable data for production of the propulsion system/ejector combination for the HSCT.

5.3: FINAL THOUGHTS

Looking back on everything that has been done so far, ejectors seem very promising for use as a thrust augmentation device on the HSCT during supersonic cruise. Because this was only a first order analysis, more can be done, and more should be done! Ejectors are already proposed for use during the takeoff phase, since they can be used for noise damping, so their possible use in the cruise configuration should not be ignored.

It is the author's recommendation that this problem be studied further in some if not all of the ways mentioned above. Then the viability of using ejectors as a thrust augmenting device in supersonic cruise can be determined definitively one way or the other.

ENDNOTES

1. See Appendix A for the data provided by the Boeing Company.

2. See page 563 of Introduction to Flight, 3[ed]. by Anderson for standard atmosphere data.

3. See pages 134-5 of Liepmann and Roshko for continuity, momentum and energy equation derivations.

4. See Appendix B for an analytic solution to M_6 as derived by Alperin and Wu from Thrust Augmenting Ejectors, Part I.

5. See page 189-192 of Aerothermodynamics ... by Oates for the derivation of the thrust equation.

6. See page 123 of Physical Gas Dynamics. by Vincenti and Krueger.

7. See page 33 of Gas Dynamics Tables for Air by Emmons.

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APPENDIX A: BOEING ENGINE DATA

According to the Boeing engine data given, the relevant information for the primary stream and the ambient conditions are, altitude, flight mach number, ambient pressure, ambient temperature, true airspeed, core stream total temperature, core stream total gas flow at mixing plane, and core stream total pressure at mixing plane.

The conditions given were:

Alt = 50,000 ft
Mach Number = 2.4
Ambient Pressure = 1.682 psia
Ambient Temperature = 390 R
True Airspeed = 2324 ft/s
Total Temperature = 2277 R
Total Gas Flow = 436.92 lbm/s
Total Pressure = 46.96 psia

All of the data, except for altitude, will be converted to SI units for ease of calculation. Since altitude is not a calculation variable, it has not been converted, and it also gives a better intuitive feel as to the flight level in most aviation related fields. Knowing that 1 atm = 14.7 psia = 101,350 Pa allows for easy conversion of the pressure terms, and temperature is easily converted since 1° K = 1.8° R. Other useful conversion factors are 1 ft = 0.3048 m and 1 lbm = 0.454 kg. Thus converting these values gives:

Alt = 50,000 ft
Mach Number = 2.4
Ambient Pressure = 11,597 Pa
Ambient Temperature = 216.7 K
True Airspeed ≈ 708 m/s
Total Temperature = 1265 K
Total Pressure = 323,780 Pa

While this data comes from the Boeing Company, the values of ambient pressure and temperature were cross checked with tables of a standard atmosphere and the value for true airspeed was calculated directly from the definition of Mach number and the given value of temperature. Also, the values of total temperature and pressure that were given for the core flow had to be used to model the engine they used.
Here is the data in raw form:

Table 10: Boeing Data Format

	ALT	MN	DTAMB	XXX	XXX	PAMB	TAMB
V0	XXX	XXX	XXX	XXX	XXX	XXX	XXX
XXX	XXX	TT6	XXX	XXX	XXX	XXX	XXX
XXX	XXX	WG8	TT8	PT8	XXX	XXX	A8GEOSTW
A9STW	WGMIX	TTMIX	PTMIX	XXX	XXX	A9MIX	XXX
VJI	XXX	XXX	XXX	XXX	XXX	XXX	XXX
XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX
XXX	XXX	XXX	XXX	XXX	NPR	A8GEOSP	
A9SF	XXX	TT16	PT16	WG16	WG55		
XXX	XXX	XXX	XXX	XXX	XXX	TT55	TT7
XXX	XXX	XXX	XXX	XXX	XXX	PT55	XXX
WS	PTS	TTS	VJSBP	AJSESP			
WSMIX	PTMIX	TTMIX	VJIM	A9MIXSP			

Table 11: Boeing Raw Data

	50000.	2.40	.0	00.00	.0	1.682	390.0
2324.	0.00	00.00	000.0	000.00	000.00	.0	000.0
0000.	000.0	2277.	00000.	00000.	0.000	.0	00.00
0000.	0000.0	701.94	1852.	46.45	0.000	000.0	1290.
795.	.0	.0	.0	.0	.0	.0	00000.
754.	0.000	.0	.0	00000.	000.00	0.000	0.000
.00	0.00	0.00	0.000	0000.	000.	0.000	00.00
0.000	.000	0000	00000.	000.00	27.62	8.958	
3.30	00.00	1079.	49.31	265.02	436.92		
.000	0.000	0.000	0000.	0000.	000.0	2277.	1852.
000.	0000.0	000.0	0.0000	0.000	000.0	46.96	0.000

The relevant data in the above tables are underlined, and were converted on the previous page. The underlined abbreviations mean in the previous table mean:

ALT - Geopotential Altitude - FT
MN - Flight Mach Number
PAMB - Ambient Pressure - PSIA
TAMB - Ambient Temperature - R
V0 - True Airspeed - FT/SEC
WG55 - Core Stream Total Gas Flow at Mixing Plane - LBM/SEC
TT55 - Core Stream Total Temperature at Mixing Plane - R
PT55 - Core Stream Total Pressure at Mixing Plane - PSIA

APPENDIX B: ALPERIN/WU SOLUTION TO THE EJECTOR PROBLEM

In the original Alperin/Wu derivation in "High Speed Ejectors" or in "Thrust Augmenting Ejectors, Part I", different terminology was used for their station locations. In this appendix, I will do the derivation in the terms of the stations previously defined in the main body of the text.

For Alperin and Wu, they used the subscripts i, p and 2 to respectively describe the ejector, the core and the mixing plane. For the purpose of this appendix, the station numbers 5, 4 and 6 will be used in place of the subscripts just described. So, just to be clear, 5 = ejector, 4 = primary (core) and 6 = mixed flow.

Alperin and Wu's derivation clearly shows the presence of the two solutions to the mixing plane Mach number. It also gives criteria for when it is possible that there will only be one root. For their derivation the following assumptions were made:

1) All fluids are compressible and thermally and calorically perfect.
2) Skin friction and blockage losses are neglected.
3) Mixing is initiated in a constant area duct at the location where the primary flow is fully expanded (primary flow pressure is equal to the local secondary flow pressure $P_4 = P_5$).
4) Complete mixing occurs in a constant cross-sectional channel.
5) Ejector surfaces are adiabatic.

Clearly, these are very similar to the assumptions previously made in the main body, and so should lead to similar results. To begin, several basic equations need to be defined. They are:

Continuity:

$$\dot{m}_6 = \dot{m}_5 + \dot{m}_4 = \rho_5 A_5 v_5 + \rho_4 A_4 v_4 = \rho_6 A_6 v_6 \tag{B1}$$

Energy:

$$\dot{m}_4 T_{04} + \dot{m}_5 T_{05} = (\dot{m}_4 + \dot{m}_5) T_{06} \tag{B2}$$

Momentum:

$$(P_5 - P_6) A_6 + \dot{m}_5 v_5 + \dot{m}_4 v_4 - (\dot{m}_5 + \dot{m}_4) v_6 = 0 \tag{B3}$$

Area Ratios:

$$\alpha = A_6 / A_4 \tag{B4}$$

and since $A_6 = A_5 + A_4$

$$(\alpha - 1) = A_5 / A_4 \tag{B5}$$

Stagnation Temperature Relations:

$$T_{05} = T_5 \left(1 + \frac{\gamma - 1}{2} M_5^2 \right) \tag{B6}$$

$$T_{06} = T_6 \left(1 + \frac{\gamma - 1}{2} M_6^2 \right) \tag{B7}$$

$$T_{04} = T_4 \left(1 + \frac{\gamma - 1}{2} M_4^2 \right) \tag{B8}$$

Acoustic Velocity:

$$v_5 = M_5 \sqrt{\gamma R T_5} \tag{B9}$$

$$v_6 = M_6 \sqrt{\gamma R T_6} \tag{B10}$$

$$v_4 = M_4 \sqrt{\gamma R T_4} \tag{B11}$$

Static Pressure:

$$P_5 = \frac{\dot{m}_4 R T_4}{v_4 A_5} = P_4 \tag{B12}$$

$$P_6 = \frac{(\dot{m}_5 + \dot{m}_4)\, R\, T_6}{v_6\, A_6} \tag{B13}$$

Static Temperature:

$$T_5 = \frac{T_{05}}{\left(1 + \frac{\gamma - 1}{2} M_5^2\right)} \tag{B14}$$

$$T_6 = \frac{T_{06}}{\left(1 + \frac{\gamma - 1}{2} M_6^2\right)} \tag{B15}$$

$$T_4 = \frac{T_{04}}{\left(1 + \frac{\gamma - 1}{2} M_4^2\right)} \tag{B16}$$

Mass Flow Ratios:

$$\mu = \frac{\dot{m}_4}{\dot{m}_5} = r = (\alpha - 1)\frac{M_5}{M_4}\sqrt{\left(\frac{T_{04}}{T_{05}}\right)\frac{1 + \frac{\gamma - 1}{2} M_5^2}{1 + \frac{\gamma - 1}{2} M_4^2}} \tag{B17}$$

It is easy to prove the right hand side of equation (B17), if one takes the first equation after the variable and substitutes in the definition of mass flow. This gives

$$\mu = \frac{\rho_5\, A_5\, v_5}{\rho_4\, A_4\, v_4} = \frac{R\, T_4\, P_5}{R\, T_5\, P_4}(\alpha - 1)\frac{M_5\sqrt{\gamma\, R\, T_5}}{M_4\sqrt{\gamma\, R\, T_4}}$$

The right hand side comes from the application of the perfect gas law, (B5), (B9) and (B11). Canceling similar variables from top and bottom, remembering the primary flow is fully expanded so $P_4 = P_5$, and applying equations (B16) and (B17) one has

$$\mu = (\alpha - 1)\frac{M_5}{M_4}\sqrt{\frac{T_4}{T_5}} = (\alpha - 1)\frac{M_5}{M_4}\sqrt{\left(\frac{T_{04}}{T_{05}}\right)\frac{1 + \frac{\gamma - 1}{2} M_5^2}{1 + \frac{\gamma - 1}{2} M_4^2}}$$

Now, with the basic definitions, one can take the momentum equation above (B3) and by expressing pressures in terms of the perfect gas law, velocity in terms of Mach number and temperature in terms of stagnation temperature, M_6 can be solved for directly.

Taking (B3) and substituting (B12) and (B13) gives:

$$\left(\frac{\dot{m}_4 \, R \, T_4}{v_4 \, A_5} - \frac{(\dot{m}_5 + \dot{m}_4) \, R \, T_6}{v_6 \, A_6} \right) A_6 + \dot{m}_5 \, v_5 + \dot{m}_4 \, v_4 - (\dot{m}_5 + \dot{m}_4) \, v_6 = 0$$

Dividing through by $\dot{m}_4$ and multiplying the A_6 back into the parentheses, gives (in view of equations (B17) and (B4))

$$\left(\frac{R \, T_4 \, \alpha}{v_4} - \frac{(1 + \mu) \, R \, T_6}{v_6} \right) + \mu \, v_5 + v_4 - (1 + \mu) \, v_6 = 0$$

Substituting (B9) - (B11) into the above, then canceling the appropriate values of T and R gives

$$\left(\frac{\alpha \sqrt{R \, T_4}}{M_4 \sqrt{\gamma}} - \frac{(1+\mu)\sqrt{R \, T_6}}{M_6 \sqrt{\gamma}} \right) + \mu M_5 \sqrt{\gamma R T_5} + M_4 \sqrt{\gamma R T_4} - (1+\mu) \, M_6 \sqrt{\gamma R T_6} = 0$$

Multiplying by $1/\sqrt{R \, \gamma}$ and applying equations (B14) - (B16) gives

$$\left(\frac{\alpha}{\gamma \, M_4} \sqrt{\frac{T_{04}}{\left(1 + \frac{\gamma - 1}{2} M_4^2\right)}} - \frac{(1+\mu)}{\gamma \, M_6} \sqrt{\frac{T_{06}}{\left(1 + \frac{\gamma - 1}{2} M_6^2\right)}} \right) + \mu M_5 \sqrt{\frac{T_{05}}{\left(1 + \frac{\gamma - 1}{2} M_5^2\right)}}$$

$$+ M_4 \sqrt{\frac{T_{04}}{\left(1 + \frac{\gamma - 1}{2} M_4^2\right)}} - (1+\mu) M_6 \sqrt{\frac{T_{06}}{\left(1 + \frac{\gamma - 1}{2} M_6^2\right)}} = 0$$

Gathering like terms and multiplying by $\dfrac{\gamma M_6 \sqrt{1+\frac{\gamma -1}{2} M_6^2}}{-(1+\mu)\sqrt{T_{06}}}$ gives

$$1+\gamma M_6^2 - \frac{\alpha \gamma}{(1+\mu)\gamma} \frac{M_6}{M_4} \sqrt{\frac{T_{04}}{T_{06}}} \sqrt{\frac{1+\frac{\gamma -1}{2} M_6^2}{1+\frac{\gamma -1}{2} M_4^2}} - \frac{\gamma M_4 M_6}{(1+\mu)} \sqrt{\frac{T_{04}}{T_{06}}} \sqrt{\frac{1+\frac{\gamma -1}{2} M_6^2}{1+\frac{\gamma -1}{2} M_4^2}}$$

$$- \frac{\mu \gamma M_5 M_6}{(1+\mu)} \sqrt{\frac{T_{05}}{T_{04}}} \sqrt{\frac{T_{04}}{T_{06}}} \sqrt{\frac{1+\frac{\gamma -1}{2} M_6^2}{1+\frac{\gamma -1}{2} M_5^2}} = 0$$

The last three terms on the left hand side all have $\dfrac{-\gamma}{(1+\mu)} \sqrt{\dfrac{T_{04}}{T_{06}}} M_6 \sqrt{1+\dfrac{\gamma -1}{2} M_6^2}$, so factoring it out gives

$$1+\gamma M_6^2 - \frac{1}{(1+\mu)} \sqrt{\frac{T_{04}}{T_{06}}} \left[\sqrt{\frac{T_{05}}{T_{06}}} \frac{\mu M_5}{\sqrt{1+\frac{\gamma -1}{2} M_5^2}} \sqrt{\frac{T_{05}}{T_{04}}} + \frac{M_4}{\sqrt{1+\frac{\gamma -1}{2} M_4^2}} \left(1+\frac{\alpha}{\gamma M_4^2}\right) \right] \cdot$$

$$\gamma M_6 \sqrt{1+\frac{\gamma -1}{2} M_6^2} = 0 \text{ (equation continued from previous line)}$$

so letting

$$J = \frac{1}{(1+\mu)} \sqrt{\frac{T_{04}}{T_{06}}} \left[\sqrt{\frac{T_{05}}{T_{06}}} \frac{\mu M_5}{\sqrt{1+\frac{\gamma -1}{2} M_5^2}} \sqrt{\frac{T_{05}}{T_{04}}} + \frac{M_4}{\sqrt{1+\frac{\gamma -1}{2} M_4^2}} \left(1+\frac{\alpha}{\gamma M_4^2}\right) \right]$$

one has

$$1+\gamma M_6^2 - J \gamma M_6 \sqrt{1+\frac{\gamma -1}{2} M_6^2} = 0$$

moving the third term of the left hand side to the right hand side and squaring both sides gives

$$1 + 2\,\gamma\, M_6^2 + \left(\gamma\, M_6^2\right)^2 = J^2\,\gamma^2\, M_6^2\left(1 + \frac{\gamma - 1}{2} M_6^2\right)$$

multiplying through on the right hand side and letting $X = \gamma\, M_6^2$ gives

$$1 + 2\,X + X^2 = \gamma\, J^2 X + X^2 J^2\, \frac{\gamma - 1}{2}$$

gathering like terms in X gives

$$1 + \left(2 - \gamma\, J^2\right) X + \left(1 - J^2\, \frac{\gamma - 1}{2}\right) X^2 = 0 \tag{B18}$$

then using the quadratic formula to solve for X, one gets

$$X = \frac{-\mathrm{B} \pm \sqrt{\mathrm{B}^2 - 4\,\mathrm{A}}}{2\,\mathrm{A}} \quad \text{if B} = 2 - \gamma\, J^2 \text{ and A} = 1 - J^2\, \frac{\gamma - 1}{2}$$

knowing $X = \gamma\, M_6^2$, solving for M_6 from the above gives

$$M_6 = \sqrt{\frac{-\mathrm{B} \pm \sqrt{\mathrm{B}^2 - 4\,\mathrm{A}}}{2\,\gamma\,\mathrm{A}}} \tag{B19}$$

However, the above derivation leaves on thing to question, what is T_{06}? According to Alperin and Wu

$$\frac{T_{06}}{T_{04}} = \frac{1 + \mu\, \frac{T_{05}}{T_{04}}}{1 + \mu} \tag{B20}$$

This is easily derived from (B2). Dividing the left and right by $T_{04}\,(\dot{m}_4 + \dot{m}_5)$; then moving switching the order of the sides gives

$$\frac{T_{06}}{T_{04}} = \frac{1}{(\dot{m}_4 + \dot{m}_5)}\left(\dot{m}_4 + \dot{m}_5 \frac{T_{05}}{T_{04}}\right)$$

then factoring out primary mass flow ($\dot{m}_4$) and rewriting everything in terms of mass flow ratio (μ) gives (B20).

So, for any given set of flow properties (M_5, M_4, T_{04}, T_{05}) and area ratio (α), there are two possible solutions to (B19). Alperin and Wu refer to these Mach numbers as $M_{2(I)}$ ($M_{6,\text{first}}$) when the negative sign of (B19) is used and $M_{2(II)}$ ($M_{6,\text{second}}$) when the positive sign is used. These are the first and second solutions that were referred to in the main paper.

As Alperin and Wu note in their paper, it is obvious that M_6 has no real value if $B^2 < 4A$ and has two real solutions if $B^2 > 4A$. These two real solutions are the subsonic and supersonic solutions to the momentum equation in the mixing plane. Alperin and Wu state the following relationship between the two Mach numbers in "Thrust Augmenting Ejectors, Part I", but they prove a similar result in and appendix of "High Speed Ejectors".

$$M_{2(I)}^2 = \frac{(\gamma - 1) M_{2(II)}^2 + 2}{2\gamma\, M_{2(II)}^2 - (\gamma - 1)} \tag{B21}$$

This is the relationship between Mach numbers across a normal shock wave. However, when $B^2 = 4A$, M_6 has only one real value, and at this condition, $M_{6,\text{first}} = M_{6,\text{second}} = 1$, since

$$J = J_c = (1/\gamma)\sqrt{2(\gamma + 1)} \text{ when } B^2 = 4A \tag{B22}$$

This is the mechanical choking which was mentioned in the main paper; however, it is not directly seen in any of the cases presented. Its occurrence in terms of the main section of the paper would be indicated by a end of mixing Mach number equal to 1 and a solver solution equal to zero. Therefore if $J = J_c$, when $B^2 = 4A$, the flow will choke and there will be only one solution.

The combination of factors to give $B^2 = 4A$ was never shown in the cases ran in the creation of the results in the main paper, but it could realistically occur. This would essentially be the case if the supersonic flow of the primary stream expanded to Mach 1 while the secondary stream was "virtually" compressed by the primary flow and therefore accelerated from a subsonic value up to Mach 1. This would result in a choked solution.

One final comment regarding the Alperin and Wu solution as compared to the method used in the main section is that they chose to fix A and then vary different parameters, yet, the results are essentially the same as the results achieved by the analysis conducted by the main paper. The results are not directly comparable, but a point by point comparison can be made, and the two results are found to match.

APPENDIX C: DIFFUSER LOSS DATA

5% Diffuser Loss Data:

First Solution:

Table 12: Table of Entropy Change 5% Diffuser Loss (1st Solution)

Values of $\frac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$2.36E^{-09}$	$-5.6E^{-17}$	$-1.0E^{-15}$	$-3.8E^{-09}$
$\mu = 1.0$	0.939361	0.872174	0.876349	0.957274
$\mu = 2.0$	1.37646	1.294843	1.23408	1.369679

Table 13: Thrust Augmentation with 5% Diffuser Loss (1st Solution)

Thrust when expanded to ambient Pressure - % Thrust increase over baseline, ~107 kN

μ/ P_r=	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.5328	0.50	0.45	0.40
	0.39	0.49	0.57	0.65	0.73	0.81	0.89	0.97	1.0000	1.05	1.13	1.22
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.68	0.79	0.81	0.77	0.66	0.48	0.21	-0.15	-0.30	-0.63	-1.25	-2.06
0.2	1.38	1.62	1.72	1.74	1.70	1.57	1.35	1.03	0.89	0.58	-0.03	-0.84
0.3	2.02	2.36	2.53	2.60	2.60	2.52	2.35	2.06	1.93	1.63	1.03	0.22
0.4	2.61	3.04	3.26	3.37	3.40	3.36	3.22	2.96	2.84	2.55	1.97	1.16
0.5	3.17	3.65	3.91	4.05	4.11	4.09	3.98	3.75	3.64	3.37	2.80	1.99
0.6	3.67	4.22	4.50	4.66	4.74	4.74	4.66	4.46	4.35	4.10	3.54	2.73
0.7	4.14	4.73	5.04	5.21	5.30	5.32	5.26	5.09	4.99	4.75	4.20	3.38
0.8	4.58	5.20	5.53	5.71	5.81	5.89	5.80	5.65	5.56	5.33	4.79	3.97
0.9	4.97	5.62	5.98	6.17	6.26	6.48	6.37	6.15	6.07	5.85	5.32	4.50
1.0	5.34	6.02	6.38	6.59	6.68	7.02	6.95	6.60	6.53	6.32	5.80	4.98
1.1	5.68	6.38	6.76	6.97	7.09	7.50	7.49	7.09	6.94	6.74	6.24	5.42
1.2	6.00	6.71	7.10	7.32	7.46	7.94	7.97	7.60	7.38	7.13	6.63	5.81
1.3	6.29	7.02	7.42	7.64	7.79	8.35	8.42	8.07	7.85	7.48	6.99	6.17
1.4	6.56	7.30	7.71	7.93	8.08	8.71	8.83	8.50	8.29	7.79	7.32	6.49
1.5	6.81	7.57	7.98	8.21	8.35	9.05	9.21	8.91	8.69	8.14	7.62	6.79
1.6	7.04	7.81	8.23	8.46	8.58	9.35	9.56	9.28	9.07	8.52	7.90	7.06
1.7	7.25	8.03	8.46	8.69	8.80	9.63	9.89	9.62	9.42	8.86	8.15	7.31
1.8	7.45	8.24	8.67	8.90	8.98	9.89	10.18	9.94	9.74	9.18	8.37	7.54
1.9	7.63	8.43	8.87	9.10	9.18	10.12	10.46	10.24	10.04	9.48	8.58	7.74
2.0	7.80	8.61	9.05	9.29	9.36	10.33	10.72	10.52	10.32	9.76	8.78	7.93

Second Solution:

Table 14: Table of Entropy Change 5% Diffuser Loss (2nd Solution)

Values of $\frac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$-3.3E^{-16}$	$-5.6E^{-17}$	$-1.0E^{-15}$	$-5.6E^{-17}$
$\mu = 1.0$	0.303354	0.87217	0.876095	0.906265
$\mu = 2.0$	-0.64911	1.294605	1.23408	1.344005

Table 15: Thrust Augmentation with 5% Diffuser Loss (2nd Solution)

Thrust when expanded to ambient Pressure - % Thrust increase over baseline, ~107 kN

μ/ P_r=	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.5328	0.50	0.45	0.40
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.22**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	1.13	1.04	1.03	1.04	1.04	1.03	1.02	1.01	1.00	0.98	0.95	0.91
0.2	2.46	2.05	1.97	1.96	1.96	1.95	1.94	1.91	1.89	1.86	1.80	1.73
0.3	3.95	3.03	2.82	2.78	2.78	2.77	2.75	2.71	2.69	2.65	2.57	2.46
0.4	5.56	4.00	3.61	3.52	3.52	3.51	3.49	3.44	3.41	3.36	3.26	3.13
0.5	7.29	4.96	4.34	4.19	4.18	4.17	4.15	4.09	4.07	4.01	3.89	3.73
0.6	9.13	5.91	5.03	4.81	4.78	4.78	4.75	4.69	4.66	4.59	4.45	4.27
0.7	11.06	6.86	5.68	5.36	5.32	5.33	5.30	5.23	5.20	5.12	4.97	4.77
0.8	13.07	7.80	6.29	5.88	5.82	5.89	5.80	5.73	5.69	5.61	5.44	5.22
0.9	15.16	8.75	6.88	6.35	6.27	6.48	6.37	6.18	6.14	6.05	5.87	5.63
1.0	17.33	9.70	7.45	6.79	6.68	7.02	6.95	6.60	6.56	6.46	6.26	6.00
1.1	19.56	10.65	8.00	7.20	7.09	7.50	7.49	7.09	6.94	6.83	6.62	6.35
1.2	21.86	11.60	8.53	7.59	7.46	7.94	7.97	7.60	7.38	7.18	6.95	6.66
1.3	24.22	12.56	9.05	7.95	7.79	8.35	8.42	8.07	7.85	7.50	7.26	6.95
1.4	26.63	13.53	9.55	8.29	8.08	8.71	8.83	8.50	8.29	7.79	7.54	7.22
1.5	29.09	14.50	10.05	8.61	8.35	9.05	9.21	8.91	8.69	8.14	7.80	7.46
1.6	31.60	15.48	10.53	8.92	8.58	9.35	9.56	9.28	9.07	8.52	8.04	7.69
1.7	34.16	16.47	11.01	9.21	8.80	9.63	9.89	9.62	9.42	8.86	8.26	7.90
1.8	36.76	17.46	11.48	9.49	8.98	9.89	10.18	9.94	9.74	9.18	8.46	8.09
1.9	39.40	18.45	11.95	9.76	9.18	10.12	10.46	10.24	10.04	9.48	8.65	8.26
2.0	42.07	19.46	12.41	10.02	9.37	10.33	10.72	10.52	10.32	9.76	8.83	8.43

10% Diffuser Loss Data:

First Solution:

Table 16: Table of Entropy Change 10% Diffuser Loss (1st Solution)

Values of $\frac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$7.81E^{-09}$	$5.55E^{-16}$	$3.33E^{-16}$	$-4.3E^{-11}$
$\mu = 1.0$	0.958441	0.879811	0.884675	0.97076
$\mu = 2.0$	1.398703	1.302486	1.257077	1.380831

Table 17: Thrust Augmentation with 10% Diffuser Loss (1st Solution)

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN

μ/ P_r=	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.5328	0.50	0.45	0.40
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.22**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.43	0.54	0.56	0.50	0.37	0.16	-0.14	-0.53	-0.69	-1.05	-1.71	-2.55
0.2	0.94	1.22	1.33	1.36	1.29	1.15	0.90	0.55	0.39	0.06	-0.59	-1.44
0.3	1.43	1.82	2.02	2.10	2.09	1.99	1.79	1.47	1.33	1.01	0.38	-0.47
0.4	1.88	2.37	2.62	2.75	2.78	2.73	2.56	2.28	2.15	1.84	1.22	0.37
0.5	2.30	2.86	3.15	3.32	3.38	3.36	3.23	2.98	2.85	2.56	1.95	1.10
0.6	2.67	3.29	3.63	3.82	3.91	3.91	3.82	3.59	3.47	3.19	2.59	1.74
0.7	3.02	3.69	4.05	4.26	4.37	4.40	4.32	4.12	4.01	3.75	3.16	2.30
0.8	3.33	4.04	4.43	4.66	4.78	4.82	4.77	4.59	4.49	4.23	3.66	2.80
0.9	3.61	4.35	4.77	5.01	5.14	5.18	5.15	5.00	4.90	4.66	4.09	3.23
1.0	3.87	4.64	5.07	5.32	5.45	5.51	5.49	5.36	5.27	5.04	4.48	3.62
1.1	4.09	4.89	5.33	5.59	5.73	5.90	5.87	5.67	5.59	5.37	4.82	3.95
1.2	4.30	5.11	5.57	5.84	5.98	6.24	6.26	5.94	5.87	5.66	5.12	4.25
1.3	4.48	5.32	5.79	6.06	6.20	6.54	6.61	6.24	6.12	5.92	5.39	4.52
1.4	4.64	5.49	5.97	6.25	6.39	6.81	6.93	6.58	6.36	6.14	5.63	4.75
1.5	4.79	5.65	6.14	6.42	6.57	7.05	7.21	6.89	6.67	6.34	5.83	4.95
1.6	4.91	5.79	6.29	6.57	6.48	7.25	7.46	7.16	6.95	6.51	6.01	5.13
1.7	5.02	5.91	6.42	6.70	6.85	7.44	7.69	7.41	7.20	6.66	6.17	5.28
1.8	5.12	6.02	6.53	6.82	6.96	7.59	7.89	7.64	7.43	6.85	6.31	5.41
1.9	5.20	6.11	6.62	6.92	7.06	7.73	8.07	7.84	7.63	7.06	6.42	5.53
2.0	5.27	6.19	6.71	7.00	7.14	7.85	8.23	8.02	7.82	7.24	6.52	5.62

Second Solution:

Table 18: Table of Entropy Change 10% Diffuser Loss (2nd Solution)

Values of $\frac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$2.78E^{-16}$	$5.55E^{-16}$	$3.33E^{-16}$	$1.11E^{-16}$
$\mu = 1.0$	0.303683	0.877634	0.881424	0.909129
$\mu = 2.0$	-0.65814	1.29975	1.257077	1.347212

Table 19: Thrust Augmentation with 10% Diffuser Loss (2nd Solution)

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN

μ/ P_r=	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.5328	0.50	0.45	0.40
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.22**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.98	0.90	0.89	0.90	0.90	0.90	0.89	0.88	0.87	0.86	0.82	0.79
0.2	2.18	1.78	1.70	1.69	1.69	1.69	1.68	1.65	1.64	1.61	1.55	1.48
0.3	3.55	2.65	2.43	2.39	2.39	2.39	2.37	2.33	2.32	2.28	2.20	2.10
0.4	5.07	3.50	3.10	3.01	3.00	3.00	2.98	2.94	2.92	2.87	2.77	2.64
0.5	6.70	4.35	3.72	3.57	3.55	3.55	3.53	3.48	3.45	3.40	3.28	3.13
0.6	8.45	5.20	4.30	4.07	4.03	4.03	4.01	3.96	3.93	3.87	3.74	3.56
0.7	10.30	6.05	4.85	4.52	4.46	4.46	4.44	4.39	4.36	4.29	4.14	3.95
0.8	12.24	6.90	5.36	4.93	4.85	4.85	4.83	4.77	4.74	4.66	4.50	4.29
0.9	14.26	7.75	5.85	5.30	5.19	5.19	5.18	5.11	5.08	4.99	4.82	4.59
1.0	16.36	8.61	6.32	5.64	5.50	5.51	5.49	5.42	5.38	5.29	5.11	4.86
1.1	18.53	9.47	6.77	5.95	5.77	5.90	5.87	5.70	5.66	5.56	5.37	5.10
1.2	20.77	10.34	7.21	6.23	6.01	6.24	6.26	5.95	5.90	5.80	5.59	5.32
1.3	23.07	11.22	7.63	6.50	6.23	6.54	6.61	6.24	6.12	6.01	5.79	5.50
1.4	25.43	12.10	8.04	6.74	6.42	6.81	6.93	6.58	6.36	6.20	5.97	5.67
1.5	27.84	12.99	8.45	6.97	6.59	7.05	7.21	6.89	6.67	6.37	6.13	5.81
1.6	30.30	13.89	8.84	7.18	6.75	7.25	7.46	7.16	6.95	6.52	6.27	5.94
1.7	32.81	14.79	9.23	7.38	6.88	7.44	7.69	7.41	7.20	6.66	6.39	6.04
1.8	35.36	15.71	9.61	7.56	7.00	7.59	7.89	7.64	7.43	6.85	6.49	6.14
1.9	37.96	16.63	9.98	7.74	7.11	7.73	8.07	7.84	7.63	7.06	6.58	6.21
2.0	40.59	17.55	10.36	7.90	7.20	7.85	8.23	8.02	7.82	7.24	6.66	6.28

15% Diffuser Loss Data:

First Solution:

Table 20: Table of Entropy Change 15% Diffuser Loss (1st Solution)

Values of $\dfrac{\Delta S_{tot}}{\dot{m}_1 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$9.99E^{-09}$	$3.33E^{-16}$	$1.44E^{-15}$	$-3.9E^{-16}$
$\mu = 1.0$	0.980067	0.890117	0.895577	0.986324
$\mu = 2.0$	1.423613	1.312721	1.282924	1.394148

Table 21: Thrust Augmentation with 15% Diffuser Loss (1st Solution)

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN, Diffuser loss=15%

μ/ Pr=	**0.90**	**0.85**	**0.80**	**0.75**	**0.70**	**0.65**	**0.60**	**0.55**	**0.5328**	**0.50**	**0.45**	**0.40**
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.20**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.12	0.25	0.26	0.18	0.03	-0.21	-0.53	-0.96	-1.14	-1.52	-2.22	-3.10
0.2	0.45	0.76	0.89	0.91	0.83	0.66	0.39	0.00	-0.16	-0.52	-1.20	-2.09
0.3	0.77	1.22	1.44	1.53	1.51	1.40	1.17	0.83	0.67	0.33	-0.34	-1.23
0.4	1.06	1.62	1.91	2.05	2.09	2.02	1.84	1.52	1.38	1.05	0.39	-0.50
0.5	1.33	1.97	2.31	2.50	2.57	2.54	2.40	2.12	1.98	1.67	1.02	0.13
0.6	1.57	2.28	2.66	2.88	2.99	2.99	2.88	2.62	2.50	2.20	1.56	0.66
0.7	1.78	2.54	2.96	3.21	3.34	3.37	3.28	3.05	2.93	2.65	2.02	1.12
0.8	1.97	2.77	3.22	3.49	3.64	3.68	3.62	3.42	3.31	3.03	2.41	1.51
0.9	2.12	2.96	3.44	3.72	3.89	3.95	3.91	3.72	3.62	3.36	2.75	1.84
1.0	2.25	3.12	3.62	3.92	4.09	4.17	4.14	3.98	3.89	3.63	3.03	2.12
1.1	2.36	3.26	3.78	4.09	4.27	4.35	4.34	4.20	4.11	3.86	3.27	2.36
1.2	2.45	3.37	3.90	4.22	4.41	4.49	4.49	4.37	4.29	4.05	3.47	2.55
1.3	2.51	3.45	4.00	4.33	4.52	4.61	4.64	4.51	4.43	4.21	3.64	2.72
1.4	2.56	3.52	4.08	4.41	4.60	4.73	4.84	4.62	4.55	4.34	3.77	2.84
1.5	2.59	3.56	4.13	4.47	4.67	4.86	5.02	4.70	4.64	4.43	3.88	2.94
1.6	2.60	3.59	4.17	4.52	4.71	4.96	5.17	4.86	4.70	4.50	3.96	3.02
1.7	2.60	3.60	4.19	4.54	4.74	5.04	5.29	5.00	4.78	4.55	4.01	3.07
1.8	2.59	3.60	4.19	4.55	4.75	5.09	5.39	5.12	4.91	4.58	4.05	3.10
1.9	2.56	3.58	4.18	4.54	4.74	5.12	5.46	5.22	5.01	4.59	4.06	3.11
2.0	2.52	3.55	4.15	4.52	4.72	5.13	5.52	5.30	5.09	4.58	4.06	3.11

Second Solution:

Table 22: Table of Entropy Change 15% Diffuser Loss (2nd Solution)

Values of $\frac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	1.11E^{-15}	3.33E^{-16}	1.44E^{-15}	-3.9E^{-16}
$\mu = 1.0$	0.30201	0.883587	0.887248	0.912787
$\mu = 2.0$	-0.67188	1.305672	1.282924	1.351426

Table 23: Thrust Augmentation with 15% Diffuser Loss (2nd Solution)

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN, Diffuser loss=15%

μ/ P_r=	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.5328	0.50	0.45	0.40
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.20**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.83	0.75	0.74	0.75	0.76	0.76	0.75	0.74	0.73	0.72	0.69	0.65
0.2	1.89	1.49	1.41	1.40	1.41	1.41	1.40	1.37	1.36	1.34	1.28	1.21
0.3	3.14	2.23	2.01	1.97	1.97	1.97	1.96	1.93	1.91	1.87	1.80	1.70
0.4	4.55	2.97	2.56	2.46	2.45	2.45	2.44	2.40	2.38	2.34	2.25	2.12
0.5	6.09	3.70	3.06	2.90	2.87	2.87	2.86	2.81	2.79	2.74	2.63	2.48
0.6	7.75	4.44	3.53	3.28	3.23	3.23	3.22	3.17	3.15	3.09	2.96	2.80
0.7	9.51	5.19	3.96	3.61	3.54	3.54	3.52	3.48	3.45	3.38	3.25	3.06
0.8	11.38	5.94	4.36	3.91	3.81	3.80	3.79	3.74	3.71	3.64	3.49	3.29
0.9	13.33	6.69	4.75	4.16	4.03	4.03	4.02	3.96	3.93	3.86	3.70	3.48
1.0	15.36	7.45	5.11	4.39	4.23	4.22	4.21	4.15	4.12	4.04	3.87	3.63
1.1	17.47	8.22	5.46	4.59	4.39	4.37	4.37	4.31	4.28	4.19	4.01	3.76
1.2	19.65	9.00	5.79	4.77	4.52	4.50	4.50	4.44	4.41	4.32	4.12	3.86
1.3	21.89	9.79	6.11	4.93	4.63	4.61	4.64	4.55	4.51	4.42	4.21	3.94
1.4	24.19	10.58	6.42	5.07	4.71	4.73	4.84	4.63	4.59	4.49	4.28	3.99
1.5	26.55	11.39	6.73	5.19	4.77	4.86	5.02	4.70	4.65	4.55	4.33	4.02
1.6	28.96	12.20	7.02	5.30	4.82	4.96	5.17	4.86	4.70	4.59	4.36	4.04
1.7	31.42	13.02	7.31	5.39	4.85	5.04	5.29	5.00	4.78	4.61	4.37	4.04
1.8	33.93	13.85	7.60	5.48	4.87	5.09	5.39	5.12	4.91	4.61	4.36	4.03
1.9	36.48	14.68	7.88	5.55	4.87	5.12	5.46	5.22	5.01	4.61	4.34	4.00
2.0	39.07	15.53	8.15	5.61	4.85	5.13	5.52	5.30	5.09	4.58	4.31	3.95

20% Diffuser Loss Data:

First Solution:

Table 24: Table of Entropy Change 20% Diffuser Loss (1st Solution)

Values of $\dfrac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$1.12E^{-08}$	$-1.2E^{-15}$	$-1.7E^{-15}$	$-6.1E^{-16}$
$\mu = 1.0$	1.004547	0.903166	0.909141	1.004222
$\mu = 2.0$	1.451494	1.325713	1.311922	1.409884

Table 25: Thrust Augmentation with 20% Diffuser Loss (1st Solution)

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN, Diffuser loss=20%

μ/ P_r=	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.5328	0.50	0.45	0.40
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.20**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	-0.24	-0.09	-0.09	-0.19	-0.36	-0.63	-0.99	-1.45	-1.64	-2.04	-2.78	-3.71
0.2	-0.11	0.24	0.38	0.40	0.31	0.12	-0.18	-0.60	-0.78	-1.16	-1.88	-2.82
0.3	0.03	0.53	0.78	0.88	0.86	0.73	0.48	0.10	-0.06	-0.43	-1.14	-2.07
0.4	0.16	0.78	1.11	1.27	1.31	1.23	1.02	0.68	0.53	0.18	-0.52	-1.45
0.5	0.27	0.98	1.38	1.59	1.67	1.63	1.47	1.16	1.02	0.68	0.00	-0.94
0.6	0.35	1.15	1.59	1.84	1.96	1.96	1.83	1.55	1.42	1.09	0.42	-0.52
0.7	0.42	1.27	1.76	2.04	2.19	2.22	2.12	1.87	1.74	1.43	0.77	-0.18
0.8	0.46	1.37	1.88	2.20	2.37	2.42	2.35	2.12	2.00	1.70	1.05	0.10
0.9	0.48	1.43	1.97	2.31	2.50	2.57	2.52	2.32	2.20	1.92	1.27	0.32
1.0	0.48	1.46	2.03	2.38	2.59	2.68	2.65	2.46	2.36	2.08	1.45	0.49
1.1	0.46	1.47	2.06	2.43	2.64	2.75	2.73	2.57	2.47	2.20	1.57	0.61
1.2	0.41	1.45	2.06	2.44	2.67	2.78	2.78	2.63	2.54	2.28	1.66	0.69
1.3	0.35	1.41	2.04	2.43	2.66	2.78	2.79	2.67	2.58	2.33	1.72	0.74
1.4	0.28	1.36	1.99	2.39	2.63	2.76	2.78	2.67	2.58	2.35	1.74	0.76
1.5	0.18	1.28	1.93	2.33	2.58	2.71	2.74	2.64	2.56	2.33	1.73	0.75
1.6	0.07	1.19	1.85	2.26	2.51	2.64	2.67	2.59	2.52	2.30	1.70	0.71
1.7	-0.05	1.08	1.75	2.17	2.42	2.55	2.67	2.52	2.45	2.24	1.65	0.65
1.8	-0.18	0.95	1.63	2.06	2.31	2.44	2.65	2.42	2.36	2.16	1.58	0.57
1.9	-0.33	0.82	1.50	1.93	2.19	2.32	2.61	2.36	2.25	2.06	1.48	0.48
2.0	-0.49	0.67	1.36	1.79	2.05	2.18	2.56	2.33	2.13	1.94	1.37	0.36

Second Solution:

Table 26: Table of Entropy Change 20% Diffuser Loss (2nd Solution)

Values of $\frac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$2.78E^{-16}$	$-1.2E^{-15}$	$-1.7E^{-15}$	$-6.1E^{-16}$
$\mu = 1.0$	0.29853	0.890359	0.893876	0.917363
$\mu = 2.0$	-0.69009	1.312635	1.311922	1.356806

Table 27: Thrust Augmentation with 20% Diffuser Loss (2nd Solution)

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN, Diffuser loss=20%

μ/ P_r=	**0.90**	**0.85**	**0.80**	**0.75**	**0.70**	**0.65**	**0.60**	**0.55**	**0.5328**	**0.50**	**0.45**	**0.40**
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.20**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.66	0.58	0.58	0.59	0.60	0.60	0.60	0.59	0.58	0.57	0.54	0.50
0.2	1.58	1.18	1.10	1.09	1.10	1.10	1.10	1.08	1.07	1.04	0.99	0.92
0.3	2.71	1.78	1.56	1.51	1.51	1.52	1.51	1.48	1.47	1.44	1.37	1.27
0.4	4.01	2.39	1.97	1.87	1.86	1.86	1.85	1.82	1.80	1.76	1.68	1.55
0.5	5.45	3.01	2.34	2.17	2.14	2.14	2.13	2.09	2.08	2.03	1.93	1.78
0.6	7.01	3.63	2.68	2.42	2.37	2.36	2.35	2.32	2.29	2.24	2.13	1.97
0.7	8.70	4.26	2.99	2.63	2.55	2.54	2.53	2.49	2.47	2.41	2.28	2.10
0.8	10.48	4.90	3.28	2.80	2.68	2.67	2.66	2.62	2.60	2.53	2.40	2.20
0.9	12.36	5.55	3.55	2.93	2.78	2.77	2.76	2.72	2.69	2.62	2.47	2.26
1.0	14.32	6.21	3.80	3.04	2.85	2.83	2.82	2.78	2.75	2.68	2.52	2.30
1.1	16.37	6.89	4.03	3.13	2.89	2.86	2.86	2.81	2.78	2.71	2.54	2.30
1.2	18.48	7.57	4.25	3.19	2.90	2.87	2.86	2.82	2.79	2.71	2.53	2.28
1.3	20.66	8.26	4.47	3.23	2.89	2.85	2.85	2.80	2.77	2.69	2.50	2.23
1.4	22.91	8.96	4.67	3.25	2.86	2.80	2.81	2.76	2.73	2.64	2.45	2.17
1.5	25.21	9.67	4.87	3.26	2.80	2.74	2.75	2.70	2.67	2.58	2.37	2.08
1.6	27.57	10.39	5.06	3.26	2.74	2.66	2.67	2.62	2.59	2.49	2.28	1.98
1.7	29.98	11.11	5.24	3.24	2.65	2.56	2.67	2.53	2.49	2.39	2.18	1.87
1.8	32.44	11.85	5.42	3.22	2.55	2.45	2.65	2.42	2.38	2.28	2.05	1.73
1.9	34.95	12.60	5.60	3.18	2.44	2.32	2.61	2.36	2.26	2.15	1.92	1.59
2.0	37.49	13.36	5.77	3.13	2.32	2.18	2.56	2.33	2.13	2.01	1.77	1.43

25% Diffuser Loss Data:

First Solution:

Table 28: Table of Entropy Change 25% Diffuser Loss (1st Solution)

Values of $\frac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$2.78E^{-09}$	$3.33E^{-16}$	$5.55E^{-16}$	$-2.3E^{-15}$
$\mu = 1.0$	1.032247	0.919235	0.925613	1.024761
$\mu = 2.0$	1.482709	1.341759	1.332471	1.428351

Table 29: Thrust Augmentation with 25% Diffuser Loss (1st Solution)

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN, Diffuser loss=25%

μ/ P_r=	**0.90**	**0.85**	**0.80**	**0.75**	**0.70**	**0.65**	**0.60**	**0.55**	**0.5328**	**0.50**	**0.45**	**0.40**
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.20**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	-0.66	-0.50	-0.51	-0.62	-0.82	-1.11	-1.50	-2.00	-2.20	-2.63	-3.41	-4.39
0.2	-0.75	-0.35	-0.20	-0.19	-0.29	-0.51	-0.83	-1.28	-1.47	-1.88	-2.64	-3.62
0.3	-0.81	-0.24	0.04	0.14	0.11	-0.03	-0.30	-0.71	-0.89	-1.28	-2.03	-3.00
0.4	-0.86	-0.16	0.21	0.39	0.42	0.33	0.11	-0.26	-0.42	-0.80	-1.53	-2.51
0.5	-0.92	-0.12	0.32	0.56	0.65	0.61	0.43	0.09	-0.06	-0.42	-1.14	-2.13
0.6	-1.00	-0.11	0.39	0.68	0.81	0.81	0.66	0.36	0.21	-0.13	-0.84	-1.83
0.7	-1.09	-0.13	0.42	0.74	0.91	0.94	0.83	0.55	0.42	0.08	-0.62	-1.61
0.8	-1.20	-0.19	0.40	0.76	0.95	1.01	0.93	0.68	0.55	0.23	-0.46	-1.46
0.9	-1.33	-0.27	0.35	0.73	0.95	1.04	0.98	0.76	0.63	0.33	-0.36	-1.36
1.0	-1.48	-0.38	0.27	0.68	0.92	1.02	0.98	0.78	0.67	0.37	-0.31	-1.32
1.1	-1.64	-0.51	0.16	0.59	0.84	0.96	0.95	0.76	0.65	0.37	-0.30	-1.31
1.2	-1.82	-0.66	0.03	0.47	0.74	0.87	0.87	0.71	0.61	0.33	-0.33	-1.35
1.3	-2.02	-0.84	-0.12	0.33	0.61	0.75	0.77	0.62	0.52	0.25	-0.40	-1.43
1.4	-2.23	-1.03	-0.30	0.16	0.45	0.61	0.63	0.50	0.41	0.15	-0.50	-1.53
1.5	-2.46	-1.24	-0.50	-0.02	0.27	0.43	0.47	0.36	0.27	0.02	-0.63	-1.66
1.6	-2.70	-1.46	-0.71	-0.23	0.07	0.24	0.29	0.19	0.11	-0.14	-0.78	-1.82
1.7	-2.96	-1.70	-0.94	-0.45	-0.14	0.03	0.09	0.00	-0.08	-0.32	-0.95	-2.00
1.8	-3.22	-1.95	-1.18	-0.68	-0.37	-0.20	-0.14	-0.21	-0.29	-0.52	-1.14	-2.20
1.9	-3.50	-2.22	-1.44	-0.94	-0.62	-0.44	-0.38	-0.44	-0.51	-0.74	-1.36	-2.42
2.0	-3.78	-2.49	-1.70	-1.20	-0.88	-0.70	-0.63	-0.69	-0.76	-0.97	-1.59	-2.66

Second Solution:

Table 30: Table of Entropy Change 25% Diffuser Loss (2nd Solution)

Values of $\frac{\Delta S_{tot}}{\dot{m}_4 R}$	$P_r = 0.9003$	$P_r = 0.70$	$P_r = 0.55$	$P_r = 0.4$
$\mu = 0.0$	$3.33E^{-16}$	$3.33E^{-16}$	$5.55E^{-16}$	$-2.3E^{-15}$
$\mu = 1.0$	0.29339	0.898159	0.901529	0.92301
$\mu = 2.0$	-0.71263	1.320856	1.331072	1.363542

Table 31: Thrust Augmentation with 25% Diffuser Loss (2nd Solution)

Thrust when expanded to ambient Pressure -
% Thrust increase over baseline, ~107 kN, Diffuser loss=25%

μ/ P_r=	**0.90**	**0.85**	**0.80**	**0.75**	**0.70**	**0.65**	**0.60**	**0.55**	**0.5328**	**0.50**	**0.45**	**0.40**
	0.39	**0.49**	**0.57**	**0.65**	**0.73**	**0.81**	**0.89**	**0.97**	**1.0000**	**1.05**	**1.13**	**1.20**
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.48	0.40	0.40	0.42	0.43	0.43	0.43	0.42	0.41	0.40	0.38	0.34
0.2	1.25	0.84	0.76	0.75	0.76	0.77	0.77	0.75	0.74	0.72	0.67	0.60
0.3	2.25	1.29	1.06	1.02	1.02	1.03	1.02	1.00	0.99	0.96	0.89	0.80
0.4	3.43	1.77	1.33	1.22	1.21	1.22	1.21	1.18	1.17	1.13	1.05	0.94
0.5	4.77	2.25	1.56	1.37	1.34	1.34	1.34	1.31	1.29	1.25	1.16	1.02
0.6	6.24	2.75	1.77	1.48	1.42	1.42	1.42	1.39	1.37	1.32	1.21	1.06
0.7	7.83	3.27	1.94	1.55	1.46	1.45	1.45	1.42	1.40	1.34	1.23	1.05
0.8	9.54	3.79	2.10	1.59	1.46	1.44	1.44	1.41	1.38	1.33	1.20	1.01
0.9	11.34	4.33	2.24	1.59	1.42	1.40	1.40	1.36	1.34	1.28	1.14	0.94
1.0	13.23	4.88	2.37	1.57	1.36	1.32	1.32	1.29	1.26	1.20	1.05	0.84
1.1	15.20	5.44	2.48	1.53	1.26	1.22	1.22	1.18	1.16	1.09	0.93	0.71
1.2	17.26	6.02	2.59	1.46	1.14	1.09	1.09	1.05	1.03	0.96	0.79	0.55
1.3	19.38	6.60	2.68	1.38	1.00	0.94	0.93	0.90	0.87	0.80	0.63	0.38
1.4	21.56	7.20	2.77	1.28	0.84	0.76	0.76	0.72	0.70	0.62	0.44	0.18
1.5	23.81	7.81	2.85	1.16	0.66	0.57	0.57	0.53	0.50	0.43	0.24	-0.03
1.6	26.12	8.43	2.92	1.04	0.46	0.36	0.36	0.32	0.29	0.21	0.02	-0.26
1.7	28.48	9.06	2.99	0.90	0.25	0.13	0.13	0.10	0.07	-0.02	-0.21	-0.51
1.8	30.89	9.70	3.06	0.75	0.03	-0.11	-0.11	-0.14	-0.17	-0.26	-0.46	-0.77
1.9	33.34	10.35	3.12	0.59	-0.21	-0.37	-0.36	-0.40	-0.43	-0.52	-0.73	-1.04
2.0	35.84	11.02	3.18	0.43	-0.45	-0.64	-0.63	-0.66	-0.69	-0.79	-1.00	-1.32